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An entropic approach to equity market integration and consumption-based capital asset pricing models

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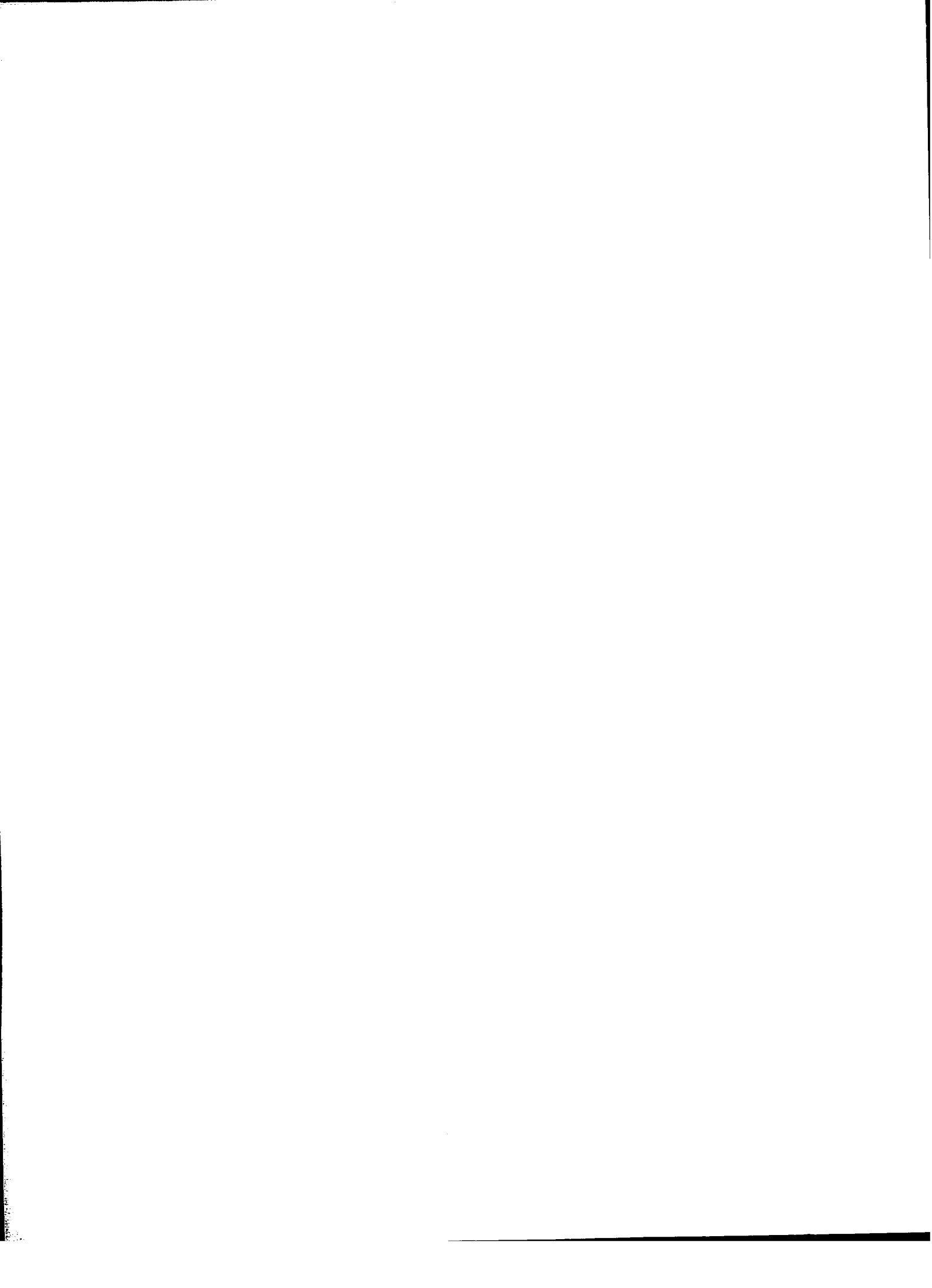
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**An entropic approach to equity market integration and consumption-based
capital asset pricing models**

by

Teng-Tsai Tu

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Major Professor: Sergio H. Lence

Iowa State University

Ames, Iowa

1998

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1 INTRODUCTION

The globalization of financial markets has been a major feature of the world economy over the last several decades. The corresponding issue of market integration among the world's equity markets has interested investors, policy-makers, and researchers for the last two decades. The question of whether international markets are integrated or not is important to investors who wish to diversify across national boundaries and to measure the risk characteristics of their portfolios. The degree of market integration is significant for a variety of public policy issues such as exchange rate behavior and the effectiveness of national monetary and fiscal policies. International asset pricing models assume either perfectly integrated or completely segmented markets. To understand the empirical relevance of such models, it is important for a researcher to know how closely integrated actual markets are. In order to deal with the practical and theoretical challenges posed by market integration, it is important that the degree of market integration be measured and understood.

According to Classens (1995) (Table 1), total portfolio flows to developing countries increased more than sevenfold from \$7.5 billions in 1989 to \$55.8 billions in 1993. As an important source of external finance for some developing countries, equity flows have been a major component of these portfolio flows. This evidence reveals the increasing importance of an emerging equity market in developing countries domestically as well as internationally. Therefore, one of the objectives of this study is to answer an interesting and important question: whether or not an emerging equity market (e.g., Taiwan) is integrated domestically. To our knowledge, this question has seldom been answered

before because the existing literature focuses mainly on the topic of international market integration.

Even in the existing literature about international financial market integration, the results and conclusions of earlier integration-related empirical studies are mixed and inconclusive (e.g., Stehle, 1977; Jorion and Schwartz, 1986; Korajczyk and Viallet, 1989; and Mittoo, 1992). Therefore, how to correctly measure the degree of market integration among a set of financial markets is still a crucial, but unsettled issue.

The major disadvantage of most earlier integration-related studies is their adoption of a parametric approach to market integration, causing the joint hypothesis test problem. To avoid the potential false rejection of the integration hypothesis due to the joint hypothesis test problem, the main purpose of the present study is to propose a nonparametric approach to measure the degree of market integration for an emerging equity market (Taiwan), as well as a mature equity market (U.S.), both domestically and internationally.

The proposed nonparametric entropic approach is based on the risk-neutral representation of the basic linear pricing rule. The risk-neutral probability measure derived from entropy pricing theory also allows us to measure the degree of market segmentation and to test whether a consumption-based capital asset pricing model is consistent or not with all assets used in an econometric analysis. Therefore, the other purpose of the present study is to propose indices to measure the degree of market segmentation domestically and internationally, and to propose an alternative entropic test of conventional consumption-based capital asset pricing models (CCAPM).

The remainder of the present dissertation is organized as follows. Chapter 2 consists of a review of three strands of literature that are related to the present study. The theoretical framework of the nonparametric entropy pricing approach is developed in chapter 3. Chapter 4 describes the portfolio return and consumption data used and the estimation method. The empirical tests of market integration and segmentation and the

alternative CCAPM test are presented in chapters 5 and 6. Finally, chapter 7 provides a summary of the conclusions from the present study and suggestions for future research.

2 LITERATURE REVIEW

Three sets of literature are related to the present study: market integration; consumption-based capital asset pricing model; and entropy pricing theory. They are reviewed separately in this chapter.

2.1 Market Integration

Two major approaches to measuring the degree of market integration have been used in the past: parametric and nonparametric approaches.

2.1.1 Parametric approach to market integration

There are two functional forms of the parametric approach to market integration: linear and nonlinear functional forms. In general, earlier studies employ the linear functional form to measure the degree of market integration, while more recent studies employ the nonlinear functional form.

Typically, previous empirical studies on international equity market integration have adopted one or more of the following international versions of the linear asset pricing models: a capital asset pricing model (e.g., Bekaert and Harvey, 1995; Errunza, Losq and Padmanabhan, 1992; Jorion and Schwarz, 1986; Errunza and Losq, 1985; and Stehle, 1977); an arbitrage pricing model (e.g., Korajczyk, 1996; Mittoo, 1992; Gultekin, Gultekin and Penati, 1989; Korajczyk and Viallet, 1989; and Cho, Eun and Senbet, 1986); and a consumption-based capital asset pricing model (e.g., Wheatley, 1988).

More recent developments of measuring market integration employ the nonlinear framework, e.g., the chaos measure of market integration (Sewell, Stansell, Lee, and Below, 1996) and the threshold autoregression model (Prakash and Taylor, 1997).

2.1.1.1 The market-based capital asset pricing model (CAPM)

In market-based asset pricing models (CAPM), the risk of an asset is measured by the covariance of its return with the return of the market portfolio. Within the CAPM framework, perfect market integration imposes restrictions between expected returns and purely domestic risk factors because the only priced risk should be the systematic risk relative to the world market portfolio. Complete market segmentation, on the other hand, implies that only domestic systematic risk should be priced. Equity market integration is typically tested by assessing whether the risk premia on a purely domestic risk factor is significantly different from zero.

Stehle (1977) uses a CAPM framework and proposes two alternative specifications to test the international equity market integration hypothesis: an integration and a segmentation specification. In his cross-sectional regression, the test for international market integration is whether or not the orthogonalized variable, i.e., purely domestic component, is significantly different from zero. The orthogonalized variable is the purely international component in the case of market segmentation. In both cases, if the orthogonalized variable is not significantly different from zero, then there is evidence for international market integration and segmentation, respectively. Stehle's estimation results are inconclusive, as neither of the two specifications can be rejected in favor of the other. The poor statistical results may indicate that these models may not be well specified. Within the context of cross-sectional regression, the lack of time variation in Stehle's estimates is another caveat.

Similar to Stehle (1977), Errunza and Losq (1985) postulate a mildly segmented market structure. The market imperfection comes from the assumed inability of a class

of investors to trade in a subset of securities because of portfolio restrictions imposed by some governments. In their models, the orthogonalized variables are residuals from a regression of the restricted and unrestricted securities on a market proxy, respectively. Their result is not inconsistent with the hypothesis that international equity markets are segmented. Errunza and Losq's results, however, are weak; they attribute this weakness to the kind of restrictions imposed in the real world. Errunza, Losq and Padmanabhan (1992) also test the same model with the more powerful maximum likelihood approach and derive results similar to Errunza and Losq's. The proxies used by Errunza and Losq (1985) are inappropriate because they use the U.S. Treasury bill rate as the riskless rate in an international context, which is usually not available in emerging markets. Furthermore, the restrictions on portfolio investments are neither necessary nor sufficient for capital markets to be segmented.

Jorion and Schwartz (1986) adopt an approach similar to Stehle (1977) to test whether the U.S. and Canadian equity markets are integrated. To correct for the thin trading of securities in the Canadian equity market, they use a Dimson (1979) estimation procedure. To make the estimation of the simultaneous equation system feasible, they greatly reduce the number of securities to nine portfolios. The results from their maximum likelihood approach strongly reject the hypothesis that the Canada and US equity markets are integrated.

The result of Mittoo (1992) confirms Jorion and Schwartz's evidence for equity market segmentation over the 1977-1981 sample period. Mittoo (1992), however, finds support for the integration hypothesis over the 1982-1986 sample period. A potential problem with Jorion and Schwartz's integration test is its reliance on their market portfolio proxy. Therefore, rejection of the integration hypothesis may simply be due to the mean-variance inefficiency of their market proxy. Another potential problem is the use of a Canadian three-month Treasury bill rate as the risk-free rate, while using stock data on a monthly basis.

Bekaert and Harvey (1995) measure the degree of market integration by an ex ante conditional probability of integration calculated from a conditional two-regime switching model. In the first (second) regime, asset returns are drawn from a completely integrated (segmented) market. An advantage of their approach is that it allows for the degree of market integration to change over time. Bekaert and Harvey (1995) find that a number of emerging markets exhibit time-varying integration.

Within the CAPM framework, the interpretation of the empirical evidence is not entirely clear. The international CAPM can only be obtained under restrictive assumptions, e.g., a universal logarithmic utility function (Adler and Dumas, 1983), purchasing power parity, or no correlation between exchange rate movements and stock returns (Solnik, 1974). There is also a problem of identifying the world market portfolio.

2.1.1.2 The multi-factor asset pricing model (APT)

In arbitrage pricing models, an asset's risk is measured by the covariance of its return with a set of systematic risk factors. Within the multi-factor APT framework, equity market integration is tested by asking whether or not the prices of risk among markets are equal. If the markets are integrated, the risk premia should be equalized through arbitrage. The null hypothesis of such tests is that the estimated prices of risk and the estimated risk-free rate are equal among equity markets. Rejection of the null hypothesis provides evidence against equity market integration.

Within the international APT framework, Cho, Eun, and Senbet (1986) test equity market integration by applying inter-battery factor analysis to stock data from eleven countries. Inter-battery factor analysis estimates the common factors between two groups of assets by examining only the inter-group's sample covariance matrix rather than the entire sample covariance matrix. From the various Chow tests, they reject three hypotheses that the risk-free rate and/or the risk premia are equal between any two countries. Therefore, they reject the hypothesis that international equity markets

are integrated. A problem with their approach is that direct comparisons of individual risk premia are not made. The individual risk premia are identified only up to an orthogonal transformation and no economic significance can be attributed to them.

Gultekin, Gultekin and Penati (1989) test for the equality in the prices of risk between Japan and U.S. equity markets using the multi-factor APT model. They use the familiar Fama and MacBeth (1973) two-stage estimation approach to carry out their integration tests. Gultekin et al. (1989) find that the hypothesis of equity market integration is rejected in the first subsample with capital controls in Japan, and not rejected in the second subsample after liberalization. This evidence supports the view that governmental policies are a source of international equity market segmentation. However, two problems arise with their test. First, they do not use some of the traditional economic factors from previous studies such as the corporate default premium or an index of industrial production because they analyze weekly data. Furthermore, most of the economic factors used in their test are not statistically significant. Second, they only test the collective risk premia, rather than the individual risk premia, from each respective country for cross-sectional differences.

Korajczyk and Viallet (1989) focus mainly on two questions. First, in comparing the explanatory power of asset pricing models, they find that multi-factor APT models tend to outperform single-factor CAPM models in both domestic and international forms. Second, they investigate the impact of changes in the regulation of financial markets on the deviations of returns from the predicted asset pricing relations. Similar to Gultekin et al. (1989), they find that the model estimates are affected by changes in capital control deregulation in international markets. Korajczyk and Viallet's study also has a problem with appropriate market proxies. Although they attempt to incorporate time variation by allowing for only three non-overlapping five-year estimates, additional time-varying estimates and tests would also be useful.

2.1.1.3 The consumption-based capital asset pricing model (CCAPM)

This part of literature will be further reviewed later. Here, we just briefly review the CCAPM model used in tests of international market integration.

In the CCAPM, an asset's risk is measured by the covariance of its return with the growth in real consumption. Wheatley (1988) tests for international equity market integration using a discrete-time version of the international CCAPM proposed by Stulz (1981). His model predicts that there will be an asset pricing line for each country that relates the expected real return facing the country's representative investor on each asset to the covariance of this return with the growth in the individual's real consumption. Based on this prediction, the hypothesis that equity markets are internationally integrated is rejected when foreign equities plot significantly off this asset pricing line. Wheatley's results provide little evidence against the international market integration hypothesis over the 1960-1985 sample period. However, his sub-period tests do not support the integration hypothesis. Wheatley's tests do not have sufficient power to reject the integration hypothesis when deviations from it are small. This is true because the standard errors of his estimates of the distances by which a foreign equity plots off a country's pricing line are large.

2.1.1.4 The nonlinear parametric approach to market integration

There are several studies that have found evidence of nonlinear dependencies in the daily, weekly and monthly changes of individual stock prices, portfolios of stocks, and exchange rates (e.g., Hsieh, 1989, 1991; Scheinkman and LeBaron, 1989; and Willey, 1992). Because of this, some researchers have used nonlinear models to test for market integration.

Based on the nonlinear framework, Sewell, Stansell, Lee and Below (1996) examine the stock market indices of five Pacific Rim countries and the U.S.. Their results of

spectral analysis tests suggest that there exist varying degrees of market integration. They also find evidence of nonlinear dependencies in some of the stock markets.

Prakash and Taylor (1997) propose a new methodology, the nonlinear threshold autoregressions, for measuring market integration in the dollar-sterling foreign exchange market. Using high-frequency data, they estimate the size of transaction-cost bands and the speed of adjustment. The changes in these measures over time provide an insight into the evolution of market integration in the foreign exchange market.

A common feature of the empirical studies reviewed thus far is that they rely on some parametric asset pricing approach. A major problem with this parametric approach is that such studies carry out a joint hypothesis test: that (i) the underlying asset pricing model is valid and that (ii) equity markets are perfectly integrated. Specifically, if the models under consideration suffer from model mis-specification, the estimated parameters are not only biased but also inconsistent, making the corresponding statistical inference invalid. Therefore, rejection of the (joint) integration hypothesis may only reflect the failure of the underlying asset pricing models. To avoid such false rejection of the integration hypothesis, a nonparametric approach to testing the integration hypothesis is needed.

2.1.2 Nonparametric approach to market integration

Hansen and Jagannathan (1992) propose a procedure that searches for a solution to the minimum (mean-square) distance between a stochastic discount factor (SDF) proxy and families of SDF's that correctly price the vector of securities used in an econometric analysis.

Benefiting on Hansen and Jagannathan's procedure, Chen and Knez (1995) propose a nonparametric approach to measure the degree of market integration among a given set of financial markets. They consider two measures of market integration, i.e., the weak and the strong integration measures corresponding to the two notions of the law of one

price (LOP) and no arbitrage opportunities, respectively. The framework of Chen and Knez (1995) is a better benchmark (relative to the parametric benchmark) for measuring the degree of integration among a set of financial markets in the sense that it is free from asset pricing model specification bias.

A major problem with the approach by Chen and Knez (1995) arises because they do not derive the sampling distribution needed to make statistical inferences about market integration. Furthermore, their calculation of weak integration measure is meaningless if it does not impose restrictions on the number of states of the world. To see this, stack the two systems of pricing equations for markets A and B. Some extra degree of freedom would arise if the number of states of the world, i.e., the unknowns, exceed the sum of numbers of assets in markets A and B, i.e., the total number of equations. In this case, we can always find a common vector of the SDF's satisfying the two systems of pricing equations simultaneously. This implies that a zero value for the weak integration measure is always guaranteed, which is meaningless to our purpose. Another problem with Chen and Knez's strong integration measure is due to inefficiency of the algorithm, in the sense that it is less efficient relative to the entropic algorithm proposed in the present study if the number of assets used is less than the number of states of the world.

2.2 Consumption-based Capital Asset Pricing Model (CCAPM)

The conventional market-based capital asset pricing model (CAPM) has two major disadvantages. First, CAPM is a static asset pricing model. It is more realistic to assume that investors simultaneously make multi-period consumption and portfolio decisions. Second, the construction of the CAPM model is based on the market portfolio, which conceptually consists of all risky assets, and is unobservable in the real world.

To attack the first problem of the market-based CAPM model, Merton (1973) developed an intertemporal multi-beta model taking into account the intertemporal nature of

the simultaneous optimal portfolio and consumption rules for an individual. Using a dynamic programming approach, Lucas (1978) derived the Euler equation. Extending the framework of Lucas (1978), Breeden (1979) first developed the single-beta consumption-based CAPM (CCAPM) model.

While investors may be heterogeneous with respect to endowments, time preferences and attitudes toward risk, Rubinstein's aggregation theorem allows us to develop a representative (composite) agent models of asset returns in which per capita consumption is perfectly correlated with the consumption stream of the typical investor.

The CCAPM model has sound microfoundations. Unfortunately, the empirical rejections of CCAPM are generally characterized by the prominent equity premium puzzle raised by Mehra and Prescott (1985) and by the Hansen-Jagannathan (1991) volatility bound tests.

2.2.1 The equity premium puzzle

The CCAPM model fails empirically mainly because of the weak correlation between consumption growth and the rate of return on stock . Specifically, over the last century, the average annual rate of return to stocks and treasury bills have been about 7% and 1%, respectively. Mehra and Prescott (1985) show that the difference in the covariance of the above two returns with consumption growth is only large enough to explain the difference in the two returns if the representative agent is implausibly highly averse to risk. The necessity of an implausibly high coefficient of relative risk aversion to explain the equity premium is referred to as the equity premium puzzle. On the other hand, the large equity premium implies that the coefficient of relative risk aversion is very high, which in turn implies that agents do not like consumption growth very much. However, although agents like consumption to be very smooth and although the risk free rate is very low, agents still save enough for per capita consumption to grow rapidly. This phenomenon is referred to as the risk-free rate puzzle (Weil, 1989).

Mehra and Prescott (1985) make three major assumptions in their model: asset markets are complete; asset markets are frictionless; and the representative agent has standard, time-additive preferences. Presumably the fit of the model to the data can be improved by relaxing one or more of those assumptions.

2.2.1.1 Modifications of standard preferences

Two major modifications have been made to the standard preferences assumed by Mehra and Prescott (1985): *generalized expected utility (GEU)* and *habit formation and consumption durability*.

i) Generalized expected utility

Under standard power-form preferences, Hall (1988) observes that the elasticity of intertemporal substitution is constrained to be equal to the reciprocal of the coefficient of relative risk aversion. He argues that this linkage is inappropriate because the former concerns the willingness of an investor to shift consumption between time periods, while the latter concerns the willingness of an investor to substitute consumption across states of the world. To break the linkage between these two different concepts, Epstein and Zin (1989, 1991) and Weil (1989) propose the *nonexpected utility* framework as a generalization of the standard preference class. The *nonexpected utility* preferences advocated by them generalize the standard time-additive expected utility specification to allow for an independent parameterization of attitudes toward risk and attitudes toward intertemporal substitution.

Empirically, Kocherlakota (1996) show that the GEU model still cannot resolve the equity premium puzzle. But it is possible to resolve the risk free rate puzzle by allowing intertemporal substitution and risk aversion to be high simultaneously.

ii) Habit formation and consumption durability

Constantinides (1990) and Sundaresan (1989) allow for adjacent complementarity in consumption, i.e., habit persistence. Intuitively, a small drop in consumption generates

a large drop in consumption net of the subsistence level, and a large drop in the marginal rate of substitution. Thus, the model generates high enough variability in the marginal rate of substitution in consumption to resolve the equity premium puzzle through habit persistence in utility and low risk aversion.

A problem with the habit persistence specification is that it is inconsistent with the notion that consumption should be relatively locally substitutable. Evidence of the existence of such consumption durability is provided by Dunn and Singleton (1986) and by Eichenbaum and Hansen (1990).

Allowing for habit formation and consumption durability simultaneously, Heaton (1995) finds that a model with short-run local substitution and long-run habit formation is consistent with the Hansen and Jagannathan (1991) volatility bounds.

2.2.1.2 Incomplete markets

If markets are incomplete, individual consumption growth will be more volatile than per capita consumption growth. As a result, individual consumption growth may covary enough with stock returns to resolve the equity premium puzzle.

Weil (1992) investigates a two-period model with incomplete financial markets. He shows that if individuals exhibit decreasing absolute prudence, the additional variability in consumption growth induced by market incompleteness helps to explain the equity premium puzzle.

Two-period models, however, cannot capture the nature of multi-period dynamic process. In an infinite horizon setting, Huggett (1993) shows numerically that the absence of income insurance markets may have little impact on lowering the risk-free rate of interest, and hence may not be able to resolve the equity premium puzzle.

The calibration experiment of Lucas (1994) demonstrates that idiosyncratic shocks to income are effectively smoothed by trading financial assets. The relationship between consumption and asset returns is similar to that predicted by the standard CCAPM

model. This conclusion is robust to several important sources of market incompleteness (e.g., short sales and borrowing constraints), which deepens the equity premium puzzle.

2.2.1.3 Market frictions

In the standard CCAPM setting, it is assumed that markets are frictionless. This unrealistic assumption implies that agents can costlessly trade any amount of the available securities. In reality, there are at least four types of market frictions: borrowing constraints; short-sales constraints; solvency constraints; and transaction costs (e.g., brokerage fees, the bid-ask spread, and taxes). He and Modest (1995) consider all four types of market frictions. They demonstrate that none of the market frictions can by itself explain the apparent failure of the CCAPM model. However, a combination of them may not be inconsistent with the conventional CCAPM model.

Heaton and Lucas (1996) show that borrowing and short sales constraints do not appear to have enough impact on the size of the equity premium to resolve this puzzle because individuals are generally constrained in both stock and bond markets.

In summary, Kocherlakota (1996) thoroughly reviews the equity premium puzzle. He concludes that this puzzle is much challenging and it is still a puzzle.

2.2.2 Volatility bound tests

Assume there are no short sales constraints, borrowing constraints, transactions costs, or other market frictions. Hansen and Jagannathan (1991) describe how to derive the admissible unconditional mean-standard deviation region, i.e., volatility bounds, for the intertemporal marginal rate of substitution (IMRS) without making any parametric assumption about utility function.

Volatility bounds can be viewed as a diagnostic tool to check whether the unconditional mean and standard deviations for the IMRS implied by alternative asset pricing models meet the admissible region implied by the asset return data. Any violation of

this volatility bound can be interpreted as a violation of the theoretical model because this volatility bound is a direct implication of such a model. If the estimated IMRS lies outside the admissible region, one might informally reject the null hypothesis that the candidate IMRS is consistent with the volatility bound.

Based on the simple comparison of the point estimates of volatility bounds with the mean and standard deviation of the IMRS implied by different preference parameterizations, many researchers have concluded that the volatility bound tests reject many commonly used utility functions for reasonable parameter values (Heaton, 1995; Ferson and Harvey, 1992; and Hansen and Jagannathan, 1991). A problem with this direct comparison is that both the estimated volatility bound and the estimated IMRS are random in nature. In other words, their tests do not take into account sampling error. The consideration of sampling error is important because the confidence regions for the parameters of the underlying asset pricing model might overlap the reasonable parameter space. Therefore, a formal test based on sampling distribution would be more appropriate.

Burnside (1994) and Cecchetti, Lam, and Mark (1994) consider the sampling error problem and develop formal statistical tests of the volatility bounds. The key question asked by Burnside (1994) is to what extent sampling errors might affect inference based on the informal (direct) comparison of point estimates. Burnside (1994) develops four asymptotic tests based on Hansen-Jagannathan bounds and uses the results of the tests to construct confidence intervals for the parameters of the CCAPM model. His results may be summarized as follows: for all test statistics, the 95% confidence regions contain part of the parameter space described by Mehra and Prescott (1985) as "reasonable". That is, they overlap with the region where the values of the rate of time preference and the coefficient of relative risk aversion are less than one and ten, respectively.

Using the generalized method of moments (GMM) procedure proposed by Hansen (1982), Cecchetti, Lam, and Mark (1994) develop and implement formal statistical tests

of the volatility bounds. Their testing strategy is based on whether the difference between these two random variables equals zero. Assuming that the consumption growth rate follows either a random walk or a first-order autoregression, they find that the failure of some asset pricing models is not nearly as extreme as the point estimates would suggest.

A major disadvantage of the above two formal tests of volatility bounds is that they are not easily applied in the presence of market frictions, as analyzed by He and Modest (1995). In this regard, Hansen, Heaton and Luttmer (1995) develop a theoretical framework for testing the volatility bounds in the presence of market frictions. Compared to the above two formal tests, Hansen, Heaton and Luttmer's tests are simpler to implement and can directly accommodate market frictions due to transaction costs or short-sale constraints.

2.3 Entropy Pricing Theory

We briefly review the concept of entropy because the present study uses the entropic approach to measure the degree of market integration and segmentation.

The entropic approach has a long history. Although it has been used in many fields, e.g., thermodynamics and image processing, this approach has not been intensively applied in economics. The entropy pricing theory, i.e., maximum-entropy principle, is consistent with the concept of informational efficiency in financial markets. The efficient market hypothesis states that informationally efficient market prices fully reflect all available relevant information. Having observed an efficient market price, all investors must be completely uncertain about the next price move. In other words, an informationally efficient price keeps all investors in the state of maximum uncertainty about the next price change. Entropy, introduced by Shannon (1948), is often used as a statistical measure of uncertainty and the negative of information. If the entropy is assumed to index

the collective market uncertainty, then the collective investors' beliefs about the future price move must be characterized by the maximum entropy (uncertainty) conditional on publicly known information that includes market prices. As a necessary condition for the informational efficiency of market prices, therefore, the maximum-entropy principle is consistent with their efficient market hypothesis.

Due to the consistency of the concept with informational market efficiency, the entropic approach has been applied to some areas of asset pricing theory. For instance, Jackwerth and Rubinstein (1996) use the entropic approach to recover risk-neutral probability distribution from European option prices. Stutzer (1996) further uses the estimated risk-neutral probabilities derived by using the maximum entropy principle to value derivative securities.

On the other hand, Robinson (1991) uses the Kullback-Leibler information criterion (KLIC) to propose a new class of tests of serial independence in time series with an application to testing the random walk hypothesis for exchange rate series. Kitamura and Stutzer (1997) develop, based on the same KLIC, a simple alternative optimally-weighted GMM framework to estimate parameters of asset pricing models. Stutzer (1995) proposes using the information bound, based on minimization of the KLIC, to diagnose asset pricing models. He gives several interpretations of the information bound (e.g., minimum distance, quasi-maximum likelihood, an optimal choice, and Bayesian) and empirically tests the small-firm effect.

3 THEORETICAL FRAMEWORK

In this chapter, we will develop a theoretical framework to measure the degree of market integration and segmentation, and to propose an alternative entropic test of the conventional CCAPM model.

The proposed theoretical framework adopts the nonparametric entropic approach, which is based on the risk-neutral representation of the basic linear pricing rule. As a building block of the entropic approach, section 1 of this chapter introduces alternative asset pricing representations. Section 2 shows the equivalence of the maximum entropy principle and the maximum likelihood estimation method through an N -dart-throwing example. The minimum cross entropy principle for testing the CCAPM model is presented in section 3. In section 4, we apply the divergence measure to test for the market integration hypothesis. Also in section 4, we propose an entropic algorithm to solve for the divergence measure. Finally, in section 5, we propose several point measures of market segmentation.

3.1 Asset Pricing Representations

It is well known that the absence of arbitrage opportunities is equivalent to the existence of some positive linear pricing rule (e.g., Ross, 1978 and Harrison and Kreps, 1979). Assume that there is a finite number of states of the world, indexed by j ($j = 1, \dots, S$), and that there is a finite number of risky assets, indexed by i ($i = 0, 1, \dots, n$).

The basic positive linear pricing rule can be represented as

$$\sum_{j=1}^S \psi_j Z_{ij} = h_i, \quad i = 0, 1, \dots, n \quad (3.1)$$

where ψ_j denotes the state price for state j that correctly prices all marketed assets, Z_{ij} denotes the gross real payoff of asset i if state j occurs, and h_i denotes the current price of asset i . The basic linear pricing rule states that the current value of an asset is the sum across states of the asset's future payoffs weighted by the corresponding state prices. Note that ψ_j is the present value of an asset that pays \$1 in state j and \$0 in all other states. Hence, ψ_j is strictly positive if there is at least one economic agent in the economy who exhibits nonsatiation.

The basic linear pricing rule (3.1) also can be represented in several equivalent ways (Ingersoll, 1987 and Dybvig and Ross, 1992). Which representation is most useful for the underlying study depends on the nature of the problem under investigation. We derive below two equivalent representations employed in the present study: the risk-neutral representation and the state-price density (or stochastic discount factor) representation.

We first derive the equivalent risk-neutral representation from the basic linear pricing rule. Multiply both sides of expression (3.1) through by a constant $\delta \equiv 1/(\sum_{j=1}^S \psi_j)$. Setting $P_j \equiv \psi_j/(\sum_{j=1}^S \psi_j)$ and rearranging terms yields :

$$\frac{1}{\delta} \sum_{j=1}^S P_j Z_{ij} = h_i, \quad i = 0, 1, \dots, n \quad (3.2)$$

or equivalently

$$\frac{1}{\delta} E^P \tilde{Z}_i = h_i, \quad i = 0, 1, \dots, n \quad (3.3)$$

where E^P denotes the expectation with respect to the probability measure P , $P \equiv (P_1, \dots, P_S)$ is called the artificial risk-neutral probability measure over states of the world, and \tilde{Z}_i denotes the random payoff of asset i . Note that $P \equiv (P_1, \dots, P_S)$ is an artificial probability measure because it is typically different from the "true" probability

measure (i.e., the "true" probability that state j will occur), but it satisfies $\sum_{j=1}^S P_j = 1$ and $P_j > 0$ for all j . If there is a risk-free asset or portfolio, then it is straightforward to show, from expression (3.2), that

$$\delta = R^f \quad (3.4)$$

where R^f denotes the gross risk-free real rate of return. Thus, the risk-neutral representation (3.3) states that the current value of an asset is the expected value of its random payoff under artificial risk-neutral probabilities discounted using the (possibly artificial) risk-free rate of return (i.e., $\delta \equiv 1/\sum \psi_j$).

Next, we derive another equivalent state-price density (or stochastic discount factor) representation from the basic linear pricing rule. Define

$$m_j = \psi_j/\Pi_j, \quad j = 1, \dots, S \quad (3.5)$$

where $\Pi_j > 0$ is the "true" probability that state j occurs. From the basic linear pricing rule (3.1) and (3.5), we obtain

$$\sum_{j=1}^S \Pi_j m_j Z_{ij} = h_i, \quad i = 0, 1, \dots, n \quad (3.6)$$

or, equivalently:

$$E^\Pi \tilde{m} \tilde{Z}_i = h_i, \quad i = 0, 1, \dots, n \quad (3.7)$$

where E^Π denotes the expectation with respect to the "true" probability measure $\Pi \equiv (\Pi_1, \dots, \Pi_S)$ and \tilde{m} is referred to as the state-price density, the stochastic discount factor (SDF), or the intertemporal marginal rate of substitution (IMRS). The state-price density or stochastic discount factor (hereafter, stochastic discount factor) representation (3.7) states that the current value of an asset is the expected value, under the true probability measure, of the asset's future random payoff multiplied by the stochastic discount factor. From the definitions of m_j and P_j , it also is straightforward to show

that

$$\delta = \frac{1}{\bar{m}} \quad (3.8)$$

where $\bar{m} \equiv \sum_{j=1}^S \Pi_j m_j$ denotes the mean of the stochastic discount factor.

The relationship between the risk-neutral representation and the stochastic discount factor representation can be characterized by the corresponding relationship between the risk neutral probability and the stochastic discount factor. Specifically, the relationship between the risk-neutral probability and the stochastic discount factor can be easily shown to be:

$$P_j = (\Pi_j / \bar{m}) m_j, \quad j = 1, \dots, S. \quad (3.9)$$

Again, expression (3.9) shows that the risk-neutral probabilities (P_j) are typically different from the "true" probabilities (Π_j). They will be equal if and only if $m_j = \bar{m} \forall j$.

One of the main purposes of the present study is to measure the degree of market integration domestically and internationally. To achieve this goal, we propose using the entropy metric to measure the degree of market integration. The derivation of the entropy metric will be based on the risk-neutral representation. The other purpose of the present study is to measure the degree of market segmentation, and to propose an alternative test of asset pricing models. These two tasks will also be based on the risk-neutral representation/probabilities.

3.2 Maximum Entropy Principle, Maximum Likelihood Estimation, and Dart Boards

It is customary to use probabilities as a measure of uncertainty about the occurrence of an event. Assume there is a finite number of states of the world, indexed by $j = 1, \dots, S$. Shannon (1948) used an axiomatic approach to define the entropy of the

unknown distribution of probabilities, $G = (G_1, G_2, \dots, G_S)$, as the metric

$$H(G) \equiv - \sum_{j=1}^S G_j \ln G_j \quad (3.10)$$

where $0 * \ln(0) = 0$. The entropy metric $H(G)$ is used to measure the uncertainty of a collection of events.

Based on the entropy concept, Jaynes (1957a,b) proposed using the maximum entropy principle to choose the unknown probability measure G . The proposed maximum entropy principle can form a basis for estimation and inference of problems for which conventional inference methods may fail to determine a unique solution.

In the present study, the distribution of probabilities to be recovered is the unknown risk-neutral probability measure P (i.e., the collective market beliefs).

3.2.1 Maximizing entropy without constraints

Assume that there is no relevant information available and that the financial market is informationally efficient. If we employ non-parametric maximum-likelihood estimation to recover a unique set of risk-neutral probabilities (i.e., the collective market beliefs), then the derived likelihood function will be exactly the same as the maximum entropy (i.e., the uncertainty index) defined by Shannon (1948). Therefore, maximizing the likelihood function is equivalent to maximizing the market entropy. The equivalence of maximizing the likelihood function and maximizing the market entropy is shown next.

The non-parametric maximum likelihood estimation is a way to choose, among all possible realizations of the risk-neutral probability measure P , the risk-neutral probability measure P^* that could have been most likely generated.

Assume that there is no relevant information available and that the financial market is informationally efficient. We can think of the risk-neutral probabilities to be recovered as the set of parameters to be estimated in the maximum likelihood procedure. In the spirit of maximum likelihood estimation, we look for the most probable realization of the

risk-neutral probability measure. To make use of the maximum likelihood method, we need to assign probabilities to various realizations of the risk-neutral probability measure. However, we do not know how agents form their subjective risk-neutral probability measures on the state space, or how the market aggregates the agents' subjective probability measures. On the other hand, we also would like to avoid any ad hoc specifications of the risk-neutral probability measure. Therefore, we assign the probabilities at random by "throwing darts" at the state space.

Suppose nature is carrying out N trials of an experiment that has S possible states. Let N_j denote the number of times that the j -th state occurs in the N -trial experiment, where

$$\sum_{j=1}^S N_j = N, \quad N_j \geq 0 \quad j = 1, \dots, S. \quad (3.11)$$

Consider the following thought experiment. Suppose nature throws a large number N of darts at the state space. Each dart falls in one of S states of the world. Let n_j denote the number of darts that fall in state j where $\sum_{j=1}^S n_j = N$. Following Gulko (1997), we first define a sample space F whose points f are all of the possible realizations of the N -dart-throwing experiment that has S possible states. The typical element of the sample space F is represented by a frequency distribution $f = (f_1, \dots, f_S)$ where

$$f_j = \frac{n_j}{N}, \quad j = 1, \dots, S \quad (3.12)$$

and

$$\sum_{j=1}^S f_j = 1. \quad (3.13)$$

Secondly, assign probabilities to every point f in the sample space F . Since there are N dart throws and each dart throw has S possible outcomes, there are S^N conceivable outcomes in the sequence of N dart throws. Suppose that all sequences are equally likely. Then each sequence occurs with probability S^{-N} . The total number of ways W

to find n_1, n_2, \dots, n_S darts in states $1, 2, \dots, S$, respectively, is the following multinomial coefficient (Feller, 1957)

$$W = W(n_1, \dots, n_S) = \frac{N!}{n_1! n_2! \dots n_S!} = \frac{N!}{\prod_{j=1}^S (n_j!)}. \quad (3.14)$$

Therefore, we assign a probability WS^{-N} to every distribution (n_1, \dots, n_S) , and consequently, to every point f in the sample space F .

Finally, we identify the most probable point f in the sample space F through a variant of the maximum likelihood approach. In order to determine the most likely point $f \in F$, we maximize over the distributions (n_1, \dots, n_S) the probability or the monotonic function of the probability

$$\ln(WS^{-N}) = -N \ln S + \ln N! - \sum_{j=1}^S \ln(n_j!). \quad (3.15)$$

Using the Stirling's approximation, $\ln x! = x \ln x - x$ as $0 < x \rightarrow \infty$, yields, for large N , the following approximation

$$\ln(WS^{-N}) \approx -N \ln S + N \ln N - \sum_{j=1}^S n_j \ln(n_j). \quad (3.16)$$

The ratio n_j/N represents the relative frequency of the possible S states in a sequence of N dart throws and

$$\frac{n_j}{N} \rightarrow P_j \quad \text{as } N \rightarrow \infty. \quad (3.17)$$

Consequently, expression (3.15) yields

$$\begin{aligned} \ln(WS^{-N}) &\approx -N \ln S + N \ln N - \sum_{j=1}^S (NP_j) \ln(NP_j) \\ &= -N \ln S - N \sum_{j=1}^S P_j \ln P_j. \end{aligned}$$

Therefore, we have

$$\ln(WS^{-N}) + N \ln S \approx -N \sum_{j=1}^S P_j \ln P_j = NH(P) \quad (3.18)$$

which is N times the Shannon entropy defined in expression (3.10).

Since both N and S are constants, we conclude that maximizing the likelihood function WS^{-N} over the frequency distribution (n_1, \dots, n_S) (i.e., maximum likelihood estimation) is equivalent to maximizing the Shannon entropy over the risk-neutral probability measure (P_1, \dots, P_S) (i.e., maximum entropy principle).

3.2.2 Maximizing entropy with constraints

Consider the risk-neutral pricing rule (3.2). The asset pricing problem amounts to finding the risk-neutral probability belief (measure) P . However, conventional asset pricing models determine the risk-neutral probability belief P by either postulating the distributions of the future asset returns (e.g., Sharpe, 1964 and Lintner, 1965) or parameterizing the distributions of the future asset prices by means of plausible stochastic processes (e.g., Black and Scholes, 1973). Apparently, each choice of return distribution or price parameterization is not equally likely. Gulko (1995) shows that not only some market beliefs are less likely than others but also many conceivable market beliefs are highly unlikely. Therefore, any ad hoc specification of the return distribution or price parameterization may determine a risk-neutral probability belief P that is relevant only on rare occasions.

3.2.2.1 The primal maximization problem with constraints

Suppose there is limited sample information in the form of the linear risk-neutral pricing constraints (3.2) available in the financial market. We divide both sides of expression (3.2) through by h_j and obtain,

$$\sum_{j=1}^S P_j R_{ij} = \delta, \quad i = 0, 1, \dots, n \quad (3.19)$$

where $R_{ij} \equiv Z_{ij}/h_i$ denotes the real gross rate of return of asset i if state j occurs and $n + 1$ denotes the total number of assets in the financial market. Subtracting the linear

pricing constraint $\sum_{j=1}^S P_j R_{0j} = \delta$ for the return of base asset 0 in the financial market from expression (3.19) yields

$$\sum_{j=1}^S P_j X_{ij} = 0, \quad i = 1, \dots, n \quad (3.20)$$

where $X_{ij} \equiv R_{ij} - R_{0j}$ denotes the relative excess rate of return of asset i (relative to the base asset return) in the financial market.

Given the above linear risk-neutral pricing constraints, how do we choose, among all possible risk-neutral probability measures, the particular set of risk-neutral probabilities that is the best estimate of the risk-neutral probability measure? In order to avoid any ad hoc specification of the risk-neutral probability measure P , a natural way to choose the optimal approximation for the risk-neutral probability measure is to choose the one, through the use of the maximum likelihood method, that is most likely to be generated and that is consistent with the given sample data. From the result of expression (3.18), following Jaynes (1957 a,b; 1984), this is the same as estimating the risk-neutral probability measure by using the maximum entropy principle with the same set of sample information (i.e., the linear risk-neutral pricing constraints) imposed. This implies that we estimate the risk-neutral probability measure P by solving the following constrained primal maximization problem:

$$\max_P H(P) = - \sum_{j=1}^S P_j \ln P_j \quad (3.21)$$

subject to

$$\sum_{j=1}^S P_j X_{ij} = 0, \quad i = 1, \dots, n \quad (3.22)$$

$$\sum_{j=1}^S P_j = 1 \quad (3.23)$$

$$P_j > 0, \quad j = 1, \dots, S \quad (3.24)$$

where expressions (3.22) and (3.23) represent the linear risk-neutral pricing constraints and the additivity constraint, respectively. Expression (3.24) imposes the positivity constraints on the risk-neutral probability measure.

To recover the risk-neutral probability measure $P \equiv (P_1, \dots, P_S)$, we form the following Lagrangian function

$$\mathcal{L} = - \sum_{j=1}^S P_j \ln P_j + \sum_{i=1}^n \lambda_i \left(- \sum_{j=1}^S P_j X_{ij} \right) + \lambda_0 \left(1 - \sum_{j=1}^S P_j \right) \quad (3.25)$$

with the first order necessary conditions

$$0 = \mathcal{L}_{P_j} = -\ln P_j - 1 - \sum_{i=1}^n \lambda_i X_{ij} - \lambda_0, \quad j = 1, \dots, S \quad (3.26)$$

$$0 = \mathcal{L}_{\lambda_i} = - \sum_{j=1}^S P_j X_{ij}, \quad i = 1, \dots, n \quad (3.27)$$

$$0 = \mathcal{L}_{\lambda_0} = 1 - \sum_{j=1}^S P_j \quad (3.28)$$

where $\lambda_i, i = 0, \dots, n$, are the Lagrange multipliers corresponding to the additivity constraint and the linear pricing rule constraints, respectively. This is a system of $S + n + 1$ equations and parameters. Solving the system yields

$$\hat{P}_j = e^{-(1+\hat{\lambda}_0)} e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}}, \quad j = 1, \dots, S \quad (3.29)$$

$$\sum_{j=1}^S (e^{-(1+\hat{\lambda}_0)} e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}}) X_{ij} = 0, \quad i = 1, \dots, n \quad (3.30)$$

$$\sum_{j=1}^S e^{-(1+\hat{\lambda}_0)} e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}} = 1, \quad i = 1, \dots, n. \quad (3.31)$$

From expression (3.31), we obtain

$$e^{-(1+\hat{\lambda}_0)} = \frac{1}{\sum_{j=1}^S e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}}} = \frac{1}{\Omega(\hat{\lambda})} \quad (3.32)$$

where $\Omega(\hat{\lambda}) \equiv \Omega(\hat{\lambda}_1, \dots, \hat{\lambda}_n) = \sum_{j=1}^S e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}}$ is a normalization factor. Substituting expression (3.32) into (3.29) yields

$$\hat{P}_j(\hat{\lambda}) = \frac{1}{\Omega(\hat{\lambda})} e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}} = \frac{e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}}}{\sum_{j=1}^S e^{-\sum_{i=1}^n \hat{\lambda}_i X_{ij}}}, \quad j = 1, \dots, S. \quad (3.33)$$

Noting that $\Omega(\hat{\lambda})$ is a constant, the value of the entropy measure H can be expressed, from expressions (3.25) and (3.33), as a function of $\hat{\lambda}$:

$$H(\hat{\lambda}) = \mathcal{L}(\hat{\lambda}) = \ln \Omega(\hat{\lambda}). \quad (3.34)$$

The primal problem of maximizing the Shannon entropy subject to moment constraints (3.22) always has a unique solution because the objective is a strictly concave function on a convex set. The solution \hat{P} satisfies the additivity constraint (3.23), and all of the \hat{P}_j are strictly positive. Observing the only remaining information for solving the Lagrange multipliers $\hat{\lambda}_j$, there is no closed-form solution for the primal problem, so that the solution must be found numerically. Nonetheless, it is possible to formulate the unconstrained dual problem to find $\hat{\lambda}$ and to increase computational efficiency (Agmon et al., 1979 and Huber, 1981).

3.2.2.2 The dual problem of the maximum entropy principle

An unconstrained dual form of the maximum entropy problem (3.21)-(3.24) was initially formulated by Agmon et al. (1979). The main purpose of the dual problem is to increase computational efficiency by solving for $\hat{\lambda}$ through the unconstrained minimization problem.

Based on the Lagrangian function (3.25) and the solution (3.33), we can write the dual objective, as a function of the Lagrange multipliers, as

$$\begin{aligned} L(\lambda) &= -\sum_{j=1}^S P_j(\lambda) \ln P_j(\lambda) - \sum_{j=1}^S P_j(\lambda) X_{ij} \\ &= \ln \Omega(\lambda) \equiv M(\lambda) \end{aligned} \quad (3.35)$$

where the optimal $P(\lambda)$ satisfies the additivity constraint. To recover $\hat{P}(\hat{\lambda})$, we can first find $\hat{\lambda}$ by solving the following unconstrained minimization problem:

$$\min_{\lambda} M(\lambda) \quad (3.36)$$

which in turn yields $\hat{P}(\hat{\lambda})$.

The following claim shows that the unconstrained minimization problem (3.36) has a unique global solution for λ .

Claim 1: The dual objective $M(\lambda)$ is strictly convex in λ and has a unique global solution for λ .

Proof: It suffices to prove that the Hessian matrix, $M_{\lambda\lambda}$, of the dual objective $M(\lambda)$ is everywhere positive definite assuring a unique global solution for λ . Note that the l -th gradient of the dual objective is

$$M_{\lambda_l} = - \sum_{j=1}^S X_{lj} P_j. \quad (3.37)$$

The l -th row, l' -th column element of the Hessian matrix of the dual objective is

$$\begin{aligned} M_{\lambda_l \lambda_{l'}} &= - \sum_{j=1}^S X_{lj} \frac{\partial P_j}{\partial \lambda_{l'}} \\ &= - \sum_{j=1}^S X_{lj} \left[\frac{1}{\Omega(\lambda)} (-X_{l'j} e^{-\sum_{i=1}^n \lambda_i X_{ij}}) + e^{-\sum_{i=1}^n \lambda_i X_{ij}} \frac{-1}{\Omega(\lambda)\Omega(\lambda)} \right. \\ &\quad \left. \sum_{j=1}^S -X_{l'j} e^{-\sum_{i=1}^n \lambda_i X_{ij}} \right] \\ &= \sum_{j=1}^S X_{lj} X_{l'j} P_j - \sum_{j=1}^S X_{lj} (P_j \sum_{j=1}^S X_{l'j} P_j). \end{aligned}$$

Since $E(\tilde{X}_i) = \sum_{j=1}^S X_{ij} P_j$ is fixed, we obtain

$$M_{\lambda_l \lambda_{l'}} = E(\tilde{X}_l \tilde{X}_{l'}) - E(\tilde{X}_l) E(\tilde{X}_{l'}) \quad (3.38)$$

$$= Cov(\tilde{X}_l, \tilde{X}_{l'}). \quad (3.39)$$

Therefore, the $n \times n$ Hessian matrix of the dual objective can be written as the following variance-covariance matrix:

$$M_{\lambda\lambda'} = \begin{bmatrix} \text{Var}(\tilde{X}_1) & \text{Cov}(\tilde{X}_1, \tilde{X}_2) & \cdots & \text{Cov}(\tilde{X}_1, \tilde{X}_n) \\ \text{Cov}(\tilde{X}_1, \tilde{X}_2) & \text{Var}(\tilde{X}_2) & \cdots & \text{Cov}(\tilde{X}_2, \tilde{X}_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(\tilde{X}_1, \tilde{X}_n) & \text{Cov}(\tilde{X}_2, \tilde{X}_n) & \cdots & \text{Var}(\tilde{X}_n) \end{bmatrix}. \quad (3.40)$$

As $P(\lambda)$ is strictly positive and the variance-covariance matrix is positive definite, the Hessian matrix $M_{\lambda\lambda'}$ is everywhere positive definite assuring a unique global solution for λ .

The minimal value of the dual objective function $M(\lambda)$ is the upper bound for the Shannon entropy of the risk-neutral probability measure $H(P)$ (i.e., $\min[M(\lambda)] = \max[H(P)]$; Alhassid et al., 1978). Furthermore, the uniqueness property of the global solutions for both primal and dual problems assures us of recovering the risk-neutral probability measure \hat{P} from $\hat{\lambda}$, where $\hat{\lambda}$ is derived from the dual problem by minimizing $M(\lambda)$ with respect to λ . This increases computational efficiency because we recover the risk-neutral probability measure \hat{P} by solving an unconstrained (dual) problem, instead of a constrained (primal) problem.

3.2.3 Economic interpretation of the maximum entropy principle

The economic interpretation of the maximum entropy principle is consistent with the informational efficiency of a financial market. An informationally efficient financial market maximizes the entropy/uncertainty of the collective market beliefs conditional on a given information set available to all agents in the market. Thus, in an informationally efficient financial market, the maximum entropy market beliefs must prevail. In other words, maximum entropy is a necessary condition for the informational efficiency of a financial market.

To justify the above interpretation, suppose that the Shannon entropy metric $H(P)$ measures the degree of the collective market uncertainty about possible asset price changes in the financial market, and attains its maximum in the state of perfect uncertainty. Then, the maximum Shannon entropy $H(P^*)$ signals that, having observed a given set of information available to all agents in the financial market, the agents as a whole have reached the state of greatest uncertainty. In this state of greatest uncertainty, any information currently available does not improve the knowledge about the next price change. In other words, the entropy maximization amounts to maximizing the collective market ignorance about any other information outside the given information set.

On the other hand, the efficient market hypothesis states that efficient market prices fully reflect all relevant information available to the agents in the financial market. In an informationally efficient financial market, all useful information is already revealed by the asset prices. Therefore, if the Shannon entropy measures the degree of market uncertainty, then the maximum entropy must be a necessary condition for the informational efficiency of the financial market. Consequently, the maximum-entropy market beliefs P^* must prevail in an informationally efficient financial market. Note that the risk-neutral probability measure P^* , derived from the maximum entropy principle, is the collective market belief because it is inferred from an information set that is available to all investors, and consequently, represents the common knowledge among the investors in the financial market.

3.3 Minimum Cross Entropy Principle for Testing CCAPM Models

In addition to the sample information (i.e., expression (3.20)) available in the financial market, suppose we also have the non-sample or prior information $q = (q_1, \dots, q_S)$ about

the unknown risk-neutral probability measure $P = (P_1, \dots, P_S)$. How do we choose the best estimate of the unknown probability measure P ? In contrast to the maximum entropy problem framework, the objective may be reformulated to minimize the entropic distance, i.e., the probabilistic distance or divergence, between the sample data in the form of P and the prior information q .

3.3.1 Minimum cross entropy principle

Following Good (1963), we choose P so as to minimize the cross entropy between the two risk-neutral probability measures P and q that is consistent with the sample information (3.20) (Gokhale and Kullback, 1978; Levine, 1980; Shore and Johnson, 1980; and Csiszar, 1991). Under the principle of minimum cross entropy, the objective is to find, out of all possible realizations of the probability measure P , the one closest to the risk-neutral probability measure q . The cross entropy principle leads to the following cross entropy minimization problem:

$$\min_P H(P | q) = \sum_{j=1}^S P_j \ln(P_j/q_j) \quad (3.41)$$

$$\sum_{j=1}^S P_j X_{ij} = 0, \quad i = 1, \dots, n \quad (3.42)$$

$$\sum_{j=1}^S P_j = 1 \quad (3.43)$$

$$P_j > 0, \quad j = 1, \dots, S \quad (3.44)$$

where the cross entropy between P and q is measured by the metric $H(P | q)$.

The minimum cross entropy principle is consistent with the maximum entropy principle. To verify this, and to simplify exposition, suppose q follows a uniform distribution,

i.e., $q_j = 1/S \forall j$. Then, the objective function of the cross entropy minimization problem can be written as

$$H(P | q) = \sum_{j=1}^S P_j \ln P_j + \ln S = -H(P) + \ln S. \quad (3.45)$$

Since S is a fixed number, minimizing the cross entropy $H(P | q)$ in (3.45) amounts to maximizing the entropy $H(P)$. Note that, similar to a Bayesian interpretation, the cross entropy minimization problem transforms the sample information and the prior probability measure q into the estimated "posterior" probability measure P^* . Under the cross-entropy principle, the deviation of the distribution of the two probability measures P and q is minimized. If the prior information q is consistent with the sample information, then $P^* = q$, and $H(P^* | q)$ has a value of 0. This means that the sample information provides no additional information relative to the prior. In general, non-trivial sample information implies that $P^* \neq q$, so that $H(P | q) > 0$. Furthermore, the larger the deviation of P from q , the larger the value of cross entropy is.

The solution to the cross entropy minimization problem (3.41) through (3.44) is analogous to the previous maximum entropy problem. The derivation of the solution to the cross entropy problem can be found in Appendix 7.

3.3.2 Hypothesis testing under cross entropy principle

Suppose we have n linear independent constraints in the form of expression (3.42), and we want to test the null hypothesis that the probability measure P is consistent with the prior q , i.e.,

$$H_o : P = q \quad \text{where} \quad \sum_{j=1}^S q_j = 1. \quad (3.46)$$

Let $H^* \equiv H(P^* | q)$ be the optimum value of the problem (3.41)-(3.44). Under the maintained assumption that the relative excess rate of returns (i.e., \tilde{X}_i) are generated by an unknown ergodic Markov chain, the estimated risk-neutral probability measure P^*

and cross entropy H^* are consistent estimators of the invariant, unknown risk-neutral probability measure P and cross entropy H , respectively (Stutzer, 1996). Under the null hypothesis (3.46), $2SH^*$ will follow asymptotically a chi-squared distribution with $(S - 1)$ degrees of freedom (Kullback, 1959; p.115), i.e.,

$$2SH^* \sim \chi_{(S-1)}^2. \quad (3.47)$$

3.3.3 An entropic test of CCAPM models

In general, any model of expected asset returns may be viewed as a model of the stochastic discount factor. Consider an agent who maximizes the expectation of a time-separable utility function:

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j U(C_{t+j}) \right] \quad (3.48)$$

where β is the time discount factor, C_{t+j} is the agent's consumption in period $t + j$, and $U(C_{t+j})$ is the one-period felicity function. At time t , the agent uses her wealth (W_t) to either consume or invest in various financial assets. That is, the agent faces the budget constraint

$$(W_t - C_t) \sum_{i=0}^n \alpha_t^i R_{t+1}^i = W_{t+1} \quad (3.49)$$

where R_{t+1}^i represents the real gross rate of return of asset i at time $t + 1$ and α_t^i denotes the fraction of the agent's investment wealth that is held in asset i at time t , and therefore $\sum_{i=0}^n \alpha_t^i = 1$. The first-order conditions or Euler equations describing the agent's optimal decision are

$$U'(C_t) = \beta E_t [U'(C_{t+1}) R_{t+1}^i], \quad i = 0, 1, \dots, n. \quad (3.50)$$

Dividing both sides of equation (3.50) by $U'(C_t)$ and taking unconditional expectations of both sides, the resulting equation yields the following unconditional equality:

$$E^\Pi [m_{t+1} R_{t+1}^i] = 1, \quad i = 0, 1, \dots, n \quad (3.51)$$

where E^Π denotes the unconditional expectations with respect to the probability measure Π , Π is the actual probability measure over states of the world, and the implied stochastic discount factor $m_{t+1} \equiv \beta U'(C_{t+1})/U'(C_t)$ is the intertemporal marginal rate of substitution between time $t+1$ and time t .

It is common in empirical literature to assume that agents can be aggregated into a "representative" agent so that we can use aggregate consumption, instead of a particular agent's consumption, to do empirical analysis. Within this context, expression (3.51) is known as the consumption-based capital asset pricing model.

Given a particular CCAPM model, how do we incorporate the CCAPM testing problem into the entropy pricing framework? Hansen and Richard (1987) point out that a candidate asset pricing model can be indexed by its implied stochastic discount factor. Analogous to Hansen and Richard (1987), we claim that a candidate asset pricing model also can be indexed by its implied risk-neutral probability measure because the risk-neutral representation is equivalent to the stochastic discount factor representation. This claim can be verified by the following analysis.

Subtracting the equilibrium condition $E^\Pi[m_{t+1}R_{t+1}^0] = 1$ for the base asset 0 from condition (3.51) yields

$$E^\Pi[m_{t+1}X_{t+1}^i] = 0, \quad i = 1, \dots, n \quad (3.52)$$

where $X_{t+1}^i \equiv R_{t+1}^i - R_{t+1}^0$ denotes the relative excess return of asset i and R_{t+1}^0 is the gross rate of return of the base asset 0. Assume there is a finite number of states of the world, indexed by j ($j = 1, \dots, S$). Expression (3.52) can be rewritten as

$$\Pi_1 m_{t+1}(1) X_{t+1}^i(1) + \Pi_2 m_{t+1}(2) X_{t+1}^i(2) + \dots + \Pi_S m_{t+1}(S) X_{t+1}^i(S) = 0, \quad i = 1, \dots, n \quad (3.53)$$

where Π_j is the actual probability that state j occurs and $m_{t+1}(j)$ is the stochastic discount factor at time $t+1$ if state j occurs. Dividing both sides of expression (3.53)

through by $\bar{m}_{t+1} = \Pi_1 m_{t+1}(1) + \Pi_2 m_{t+1}(2) + \dots + \Pi_S m_{t+1}(S)$ and ignoring the time subscript, we obtain the following expression:

$$P_1 X^i(1) + P_2 X^i(2) + \dots + P_S X^i(S) = 0, \quad i = 1, \dots, n \quad (3.54)$$

or, equivalently:

$$E^P(\tilde{X}^i) = 0, \quad i = 1, \dots, n \quad (3.55)$$

where

$$P_j \equiv \Pi_j m(j) / \bar{m}, \quad j = 1, \dots, S. \quad (3.56)$$

Therefore, a CCAPM model also can be indexed by its implied risk-neutral probability measure.

The next question is how to implement an alternative CCAPM test within the entropy pricing framework. In the present study, we propose the following two-step testing procedure to carry out an alternative CCAPM test:

Step 1: Verify the integration assumption of the CCAPM model

One of the important assumptions of conventional CCAPM models is that all assets used in an econometric analysis are traded in a perfectly integrated market. If the group of assets used in a test violates the above (implicit) assumption, the CCAPM model is destined to fail. To avoid such a false rejection of the CCAPM model, which is due to the internal pricing inconsistency inherent in the asset data set, one first needs to verify whether all assets used in the econometric analysis are traded in a perfectly integrated market.

If the asset data set passes the integration test, then one can confidently accept or reject a candidate CCAPM model, based on its performance in explaining the asset data set used in the test. Otherwise, if it fails, any inferences about the validity of the candidate CCAPM model will be biased. In the present study, the integration test will be performed via the method discussed previously in section 3.4.2.

Step 2: Implement an entropic test of the CCAPM model

Suppose that the asset data set passes the integration test of step 1 and that the sample information contained in this data set can be transformed into n linear pricing constraints (3.22). The basic logic behind the proposed alternative CCAPM test can be described as follows: If one can find a risk-neutral probability measure used to price all assets in a perfectly integrated market such that it also is statistically consistent with the risk-neutral probability measure implied by the candidate CCAPM model, then the candidate CCAPM model holds for the integrated market. Otherwise, the candidate CCAPM model fails because the candidate CCAPM model cannot explain the actual asset data set.

Formally, let $q = (q_1, \dots, q_S)$ be the risk-neutral probability measure implied by a candidate CCAPM model, and $P = (P_1, \dots, P_S)$ be the risk-neutral probability measure that prices all assets in an integrated market. We can view the probability measure q implied by a candidate CCAPM model as the non-sample or prior information about the unknown risk-neutral probability measure P used to price all assets in the market. Then we can apply the cross entropy minimization problem (i.e., (3.41)-(3.44)) to test the candidate CCAPM model.

Under the null hypothesis that the probability measure P is consistent with the probability measure q implied by the candidate CCAPM model, i.e., $H_0 : P = q$, $2SH^*$ will follow asymptotically a chi-squared distribution with $(S - n - 1)$ degrees of freedom:

$$2SH^* \sim \chi_{(S-n-1)}^2. \quad (3.57)$$

Based on this chi-squared distribution, if the null hypothesis that the two probability measures P and q are consistent is rejected, the candidate CCAPM model fails. Otherwise, it is not rejected.

The proposed testing procedure in the present study differs from that of Stutzer (1995). While the test of Stutzer (1995) is based on the minimized value of the objective

function, i.e., the information bound, the test in the present study is based on the minimizer of the minimization problem, i.e., the risk-neutral probability measure.

3.4 The Divergence Problem for Testing the Integration Hypothesis

Most tests of market integration in the existing literature rely on a parametric approach, which implies a joint hypothesis test problem. To free from the asset pricing model specification bias, Chen and Knez (1995) developed a nonparametric approach to test for market integration. The major disadvantage of their nonparametric approach is that it lacks the sampling distribution needed to make statistical inferences about market integration. To circumvent the sampling distribution problem, we resort to a new tool, entropy pricing theory. We adopt the entropic approach to test the market integration hypothesis because it is consistent with the concept of informational efficiency in financial markets.

3.4.1 Alternative representations of the perfect integration hypothesis

We will first introduce the basic concept and definition of perfect integration used in the present study, followed by alternative representations of the perfect integration hypothesis.

Each financial market has its own pricing structure to price all assets in the market. The pricing structure of a financial market can be characterized and completely summarized by its implied pricing functionals (i.e., the state prices in expression (3.1) or the stochastic discount factors in expression (3.7)). Under certain conditions, according to the Riesz representation theorem (see Luenberger, 1969), each pricing functional also can be uniquely represented by a stochastic discount factor. Therefore, the pricing structure of a financial market also can be represented by a set of stochastic discount factors. The

absence of arbitrage opportunities implies that the set of stochastic discount factors is nonempty.

Suppose there are two sets of stochastic discount factors, M_A and M_B , corresponding to the pricing structures of any two financial markets A and B , respectively. How do we determine whether or not these two financial markets are perfectly integrated? We use the following basic intuition to introduce the concept of perfect integration defined in the present study.

Given any two constructed portfolios that have identical payoffs, one from each market, if the two financial markets are perfectly integrated, then they must assign the same prices to these two portfolios. This means that the two markets must use the same pricing functional or stochastic discount factor to price all assets in the market, which in turn implies that they must have at least one stochastic discount factor in common in their pricing structures. Therefore, we can define perfect integration between any two financial markets A and B as follows:

Definition 1: Two markets A and B are said to be perfectly integrated if $M_A \cap M_B \neq \emptyset$, where \emptyset denotes the empty set (Chen and Knez, 1995).

The above definition of perfect integration is depicted in Figure 3.1. Under perfect integration, the distance between the two sets of pricing structures is equal to zero. i.e., $D(M_A, M_B) = 0$. Note that perfect market integration does not require the two sets M_A and M_B to be equal.

On the other hand, if two markets are segmented, then there is no common stochastic discount factor in their pricing structures. In this case, these two markets will use different pricing functionals to price the assets in their own markets. The degree of segmentation should ultimately be reflected by how different they use their pricing functionals or stochastic discount factors. Therefore, we can use the minimum distance between the respective sets of stochastic discount factors for the two markets to measure the degree of market segmentation. The concept of market segmentation is depicted in

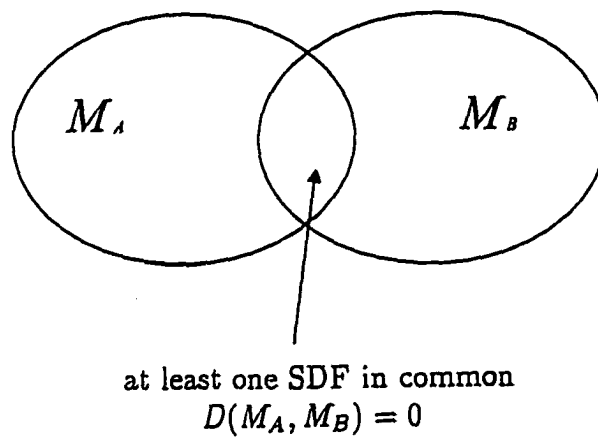


Figure 3.1 Perfect market integration

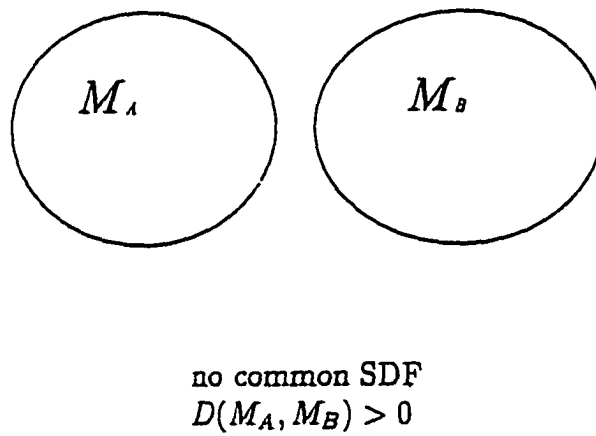


Figure 3.2 Market segmentation

Figure 3.2. Under market segmentation, the distance between the two sets of stochastic discount factors is positive. The smaller (larger) the distance between the respective sets of stochastic discount factors for the two markets, the more closely integrated (segmented) the two markets are.

The absence of arbitrage guarantees the existence of a strictly positive stochastic discount factor in the pricing structure of a financial market. However, conventional asset pricing theory offers no guidance to construct a unique positive stochastic discount factor for a financial market. Entropy pricing theory allows us to extract a unique positive stochastic discount factor for a financial market because the optimization problem of the entropic framework automatically imposes the positivity constraint on the stochastic discount factor and its solution is unique.

In order to adopt the entropic approach, the estimation of the degree of market integration between any two markets is required to be based on the risk-neutral representation, instead of the stochastic discount factor representation. Under the null hypothesis of perfect integration between the two markets A and B , the stochastic discount factor of market A should be equal to that of market B for all states of the world. In other words, an equivalent expression of the perfect integration hypothesis can be written as

$$m_j^A = m_j^B, \quad j = 1, \dots, S. \quad (3.58)$$

We argue, in the following section, that testing of the perfect integration hypothesis in terms of the equivalence of the stochastic discount factors between these two markets is equivalent to testing the same hypothesis in terms of the equivalence of the risk-neutral probabilities between these two markets. This justifies the appropriateness of using entropy pricing theory - which is based on the risk-neutral representation - to test the market integration hypothesis.

From expression (3.9), if the stochastic discount factors for the two markets A and

B are equal for each state, then the risk-neutral probabilities for the two markets A and B are equal. That is,

$$m_j^A = m_j^B \quad \forall j \Rightarrow P_j^A = P_j^B \quad \forall j. \quad (3.59)$$

Note that the converse of expression (3.59) is, in general, not true because the means of the two stochastic discount factors need not be equal. Under the null hypothesis of perfect integration, however, the means of the two stochastic discount factors should be equal, and hence the following converse expression (3.60) also is true, implying that the test based on the equivalence of two stochastic discount factors is equivalent to the test based on that of two risk-neutral probability measures.

$$P_j^A = P_j^B \quad \forall j \text{ and } \bar{m}^A = \bar{m}^B \Rightarrow m_j^A = m_j^B \quad \forall j \quad j = 1, \dots, S. \quad (3.60)$$

Therefore, another definition of perfect integration between any two financial markets A and B is given by the following:

Definition 2: Two markets A and B are said to be perfectly integrated if $\mathcal{P}_A \cap \mathcal{P}_B \neq \emptyset$, where \emptyset denotes the empty set and \mathcal{P}_A (\mathcal{P}_B) is the set of admissible risk-neutral probability measures corresponding to the pricing structure of financial market A (B).

In the present study, we use *definition 2* of perfect integration to test for the integration hypothesis.

3.4.2 Testing the perfect integration hypothesis

Analogous to Chen and Knez (1995), under the risk-neutral representation the degree of market integration and segmentation can be measured by the probabilistic distance between the two sets of admissible probability measures \mathcal{P}_A and \mathcal{P}_B for the two financial markets A and B . The smaller (larger) the probabilistic distance between the two sets of admissible risk-neutral probability measures for the two markets, the more closely

integrated (segmented) the two markets are. A zero value for the probabilistic distance between these two sets indicates perfect integration between these two markets. Entropy pricing theory allows us to measure the probabilistic distance (i.e., the divergence) between the two sets of admissible risk-neutral probability measures for the two markets, and hence the degree of market integration and segmentation between these two markets.

Let \mathcal{P}_A and \mathcal{P}_B be the two sets of admissible risk-neutral probability measures corresponding to the pricing structures of any two financial markets A and B , respectively. Suppose that there are $n_A + 1$ and $n_B + 1$ assets in these two financial markets, respectively. The sample information contained in the these two sets of assets can be transformed into the form of n_A and n_B linearly independent pricing constraints (3.22) for the two markets, respectively. Given the above two sample information sets, how do we simultaneously choose, among all possible realizations of the two risk-neutral probability measures, P^{A*} and P^{B*} for the two markets A and B that are closest to each other?

In contrast to the cross-entropy minimization problem, the corresponding divergence problem may be reformulated to minimize the divergence between the two risk-neutral probability measures P^A and P^B that are consistent with the linear pricing constraints and the additivity and positivity constraints. Specifically, we measure the divergence between the two risk-neutral probability measures P^A and P^B for the two financial markets by solving the following minimization problem:

$$\min_{P^A \in \mathcal{P}_A, P^B \in \mathcal{P}_B} D(P^A, P^B) = \sum_{j=1}^S (P_j^A - P_j^B) \ln(P_j^A / P_j^B) \quad (3.61)$$

subject to

$$\sum_{j=1}^S P_j^k X_{ij}^k = 0, \quad i = 1, \dots, n_k; k = A, B \quad (3.62)$$

$$\sum_{j=1}^S P_j^k = 1, \quad k = A, B \quad (3.63)$$

$$P_j^k > 0, \quad j = 1, \dots, S; k = A, B \quad (3.64)$$

where the metric $D(P^A, P^B)$ measures the probabilistic distance (i.e., the divergence) between the two risk-neutral probability measures P^A and P^B , k denotes the market index, and \mathcal{P}_A and \mathcal{P}_B are the two sets of admissible probability measures for P^A and P^B , respectively. Expressions (3.62) and (3.63) represent the linear pricing constraints and the additivity constraint, respectively. Expression (3.64) imposes the positivity constraints on the risk-neutral probability measures. In the present study, we use the resulting divergence metric $D^* \equiv D(P^{A^*}, P^{B^*})$ to measure the degree of market integration between the two markets A and B , where

$$(P^{A^*}, P^{B^*}) \equiv \arg \min_{P^A, P^B} (3.61) \quad \text{subject to} \quad (3.62) - (3.64).$$

A zero value of the metric $D(P^{A^*}, P^{B^*})$ indicates perfect integration between these two markets. A positive value of the metric $D(P^{A^*}, P^{B^*})$ implies that the two markets are segmented.

In the present study, we also use the concept of the equivalence of the risk-neutral probability measures between any two financial markets to test for the market integration hypothesis. According to *definition 2* of perfect market integration, the testable null hypothesis that the probability measure P^A is consistent with P^B can be formed as follows:

$$H_0 : P^A = P^B = P^0 \quad \text{where} \quad \sum_{j=1}^S P_j^0 = 1. \quad (3.65)$$

Under the null hypothesis (3.65), $(S/2)D^*$ approaches an asymptotic chi-squared distribution with $(S - n_A - n_B - 1)$ degrees of freedom (Kullback, 1959; p.130):

$$(S/2)D^* \sim \chi_{(S-n_A-n_B-1)}^2. \quad (3.66)$$

We use this chi-squared distribution to test the perfect integration hypothesis. If the null hypothesis of perfect integration is rejected, there is sufficient evidence indicating that

the two financial markets A and B are not perfectly integrated. Otherwise, we conclude (with possible type II error) that the two financial markets are perfectly integrated.

3.4.3 The entropic algorithm for solving the divergence problem

The previous divergence problem requires us to find a minimum probabilistic distance between the two convex and closed sets \mathcal{P}_A and \mathcal{P}_B . Since the objective is also convex, the minimum probabilistic distance between the two sets of admissible probability measures \mathcal{P}_A and \mathcal{P}_B is unique. However, solving the divergence problem is difficult because for two general convex and closed sets there may exist more than one solution, even though the minimum probabilistic distance between them is unique.

Following the strategy of Chen and Knez (1995), we propose the entropic algorithm to numerically solve the above divergence problem. The basic logic behind the entropic algorithm is depicted in Figure 3.3. In general, the entropic algorithm involves two iterative mapping steps. In the first step, we find the minimum probabilistic distance between a point in \mathcal{P}_A , say \tilde{P}^A , and the set \mathcal{P}_B through the use of the minimum cross-entropy principle. Let \tilde{P}^B be the probability measure in \mathcal{P}_B whose probabilistic distance from \tilde{P}^A gives the minimum probabilistic distance between \tilde{P}^A and \mathcal{P}_B . In the second step, we project the nearest map \tilde{P}^B back onto \mathcal{P}_A to find the minimum probabilistic distance between \tilde{P}^B and \mathcal{P}_A through the use of the minimum cross-entropy principle. Let $\tilde{P}^{A'}$ be the nearest map from \tilde{P}^B to \mathcal{P}_A . Restarting from $\tilde{P}^{A'}$, we repeat these two steps back and forth and stop for some prespecified stopping rule.

Let I be the number of iterations; S be the number of states of world; $\tilde{P}^k(I)$ be the probability measure in the admissible set \mathcal{P}_k used for the I -th iteration; $\tilde{\lambda}^k$ be the vector of Lagrange multipliers on the sample information (i.e., the linear pricing constraints). The entropic algorithm for calculating the minimum probabilistic distance between the two probability measures P^A and P^B can then be summarized as follows:

Step 0: Set the tolerance for the divergence (i.e., the minimum probabilistic distance)

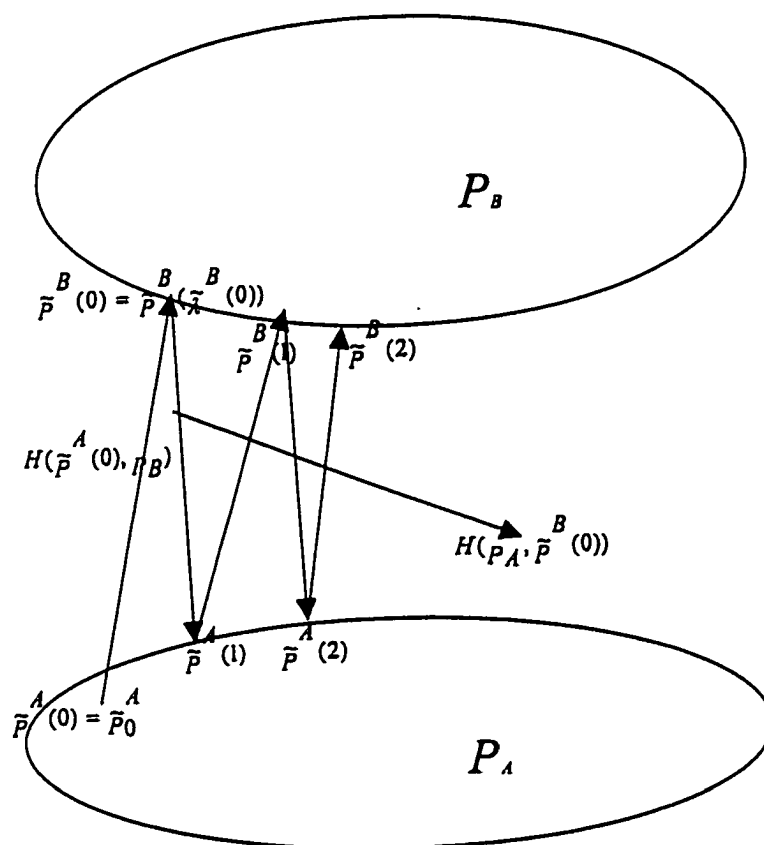


Figure 3.3 The solution strategy of the entropic algorithm

between the two probability measures P^A and P^B for the iterative mapping process. For the first iteration, also set $I = 0$ and $\tilde{P}^A(I) = \tilde{P}^{A*}$ where $\tilde{P}^{A*} = 1/S$ can be viewed as the initial prior information for $\tilde{P}^B(I)$ in the first iteration.

Step 1: Compute $\tilde{\lambda}^B(I) = \lambda_B(\tilde{P}^A(I))$, $\tilde{P}_j^B(I) = P_{B_j}(\tilde{P}^A(I))$ and $D_B(\tilde{P}^A(I), \mathcal{P}_B)$ according to the following three equations:

$$\lambda_B(\tilde{P}^A) = \arg \min_{\tilde{\lambda}^B} \Omega(\tilde{\lambda}^B) \quad (3.67)$$

$$P_{B_j}(\tilde{P}^A) = \frac{\tilde{P}_j^A e^{\sum_{i=1}^{n_B} \lambda_{B_i} x_{ij}^B}}{\Omega(\lambda_B(\tilde{P}^A))} \quad (3.68)$$

$$D_B(\tilde{P}^A, \mathcal{P}_B) = \sum_{j=1}^S (\tilde{P}_j^A - P_{B_j}(\tilde{P}^A)) \ln(\tilde{P}_j^A / P_{B_j}(\tilde{P}^A)) \quad (3.69)$$

where $\Omega(\tilde{\lambda}^B) \equiv \Omega(\lambda_{B_1}, \dots, \lambda_{B_{n_B}}) = \sum_{j=1}^S \tilde{P}_j^A e^{\sum_{i=1}^{n_B} \lambda_{B_i} x_{ij}^B}$; λ_{B_i} is the i -th element of the vector $\lambda_B(\tilde{P}^A)$; $\tilde{P}_j^A(\tilde{P}_j^B)$ is the j -th element of the probability measure $\tilde{P}^A(\tilde{P}_B)$; $P_B(\tilde{P}^A) \equiv (P_{B_1}(\tilde{P}^A), \dots, P_{B_{n_B}}(\tilde{P}^A))$ is the nearest point map from \tilde{P}^A to the set \mathcal{P}_B ; and $D(\tilde{P}^A, \mathcal{P}_B)$ is always the minimum probabilistic distance between the point \tilde{P}^A and its nearest map $P_B(\tilde{P}^A)$.

Step 2: Compute $\tilde{\lambda}^A(I) = \lambda_A(\tilde{P}^B(I))$, $\tilde{P}_j^A(I+1) = P_{A_j}(\tilde{P}^B(I))$, and $D(\mathcal{P}_A, \tilde{P}^B(I))$ with the appropriate interchanges between the indices A and B .

Step 3: Let $I = I + 1$.

Step 4: Repeat steps 1 through 3, and stop when the tolerance for the divergence between the two measures P^A and P^B is achieved.

3.5 Point Measures of Market Segmentation

Based on the risk-neutral probability measures P^{A*} and P^{B*} estimated from the divergence problem (3.61)-(3.64), entropy pricing theory also allows us to measure the degree of market segmentation between these two markets A and B .

To measure the degree of market segmentation between the two markets A and B , we first need to recover the (artificial) risk-free rate of returns for the two markets A and B from expression (3.19) by calculating

$$\delta^{k*} = \sum_{j=1}^S P_j^{k*} R_{0j}^k, \quad k = A, B \quad (3.70)$$

where P_j^{k*} is the minimizer of the divergence problem (3.61)-(3.64).

We use the cross market pricing error to measure the degree of market segmentation. Assume that $P^{k*} \equiv (P_1^{k*}, \dots, P_S^{k*})$ and that δ^{k*} correctly prices all assets in market k . One of the possible ways to measure the cross market pricing error of the i -th asset in market k_1 mispriced by the risk-neutral probability measure P^{k_2*} in market k_2 is the difference between the correct price δ^{k_1*} and the "wrong" price $\hat{\delta}_i^{k_1*}$ where δ^{k_1*} is calculated from expression (3.70) and

$$\hat{\delta}_i^{k_1*} \equiv \sum_{j=1}^S P_j^{k_2*} R_{ij}^{k_1}, \quad i = 0, 1, \dots, n_{k_1}. \quad (3.71)$$

Therefore, we can use either of the following two expressions to measure the degree of cross market pricing error of assets in market A mispriced by the "wrong" risk-neutral probability measure P^{B*} in market B :

$$PEA_1 = \sqrt{\frac{\sum_{i=0}^{n_A} (\delta^{A*} - \hat{\delta}_i^{A*})^2}{n_A}} \quad (3.72)$$

$$PEA_2 = \frac{\sum_{i=0}^{n_A} |\delta^{A*} - \hat{\delta}_i^{A*}|}{n_A} \quad (3.73)$$

where δ^{A*} is calculated from expression (3.70) and

$$\hat{\delta}_i^{A*} \equiv \sum_{j=1}^S P_j^{B*} R_{ij}^A, \quad i = 0, 1, \dots, n_A. \quad (3.74)$$

Similarly, the degree of cross market pricing error of assets in market B mispriced by the "wrong" risk-neutral probability measure P^{A*} in market A can be measured by

PEB_1 or PEB_2 with appropriate interchanges between markets A and B in expressions (3.72) and (3.73), respectively. The cross market pricing error measure approaches zero as the risk-neutral probability measure of another market approaches that of its own market, and it is zero if the two markets are perfectly integrated.

The cross market pricing error measure for market A is, in general, not equal to that for market B . Therefore, we propose several point measures to measure the degree of market segmentation between the two markets A and B . The first pair of the market segmentation indices are defined as the following two expressions by pooling the cross market errors for the two markets:

$$\begin{aligned} S_1 &= \sqrt{\frac{\sum_{i=0}^{n_A} (\delta^{A*} - \hat{\delta}_i^{A*})^2 + \sum_{i=0}^{n_B} (\delta^{B*} - \hat{\delta}_i^{B*})^2}{n^A + n^B}} \\ &= (wa * PEA_1^2 + wb * PEB_1^2)^{\frac{1}{2}} \end{aligned} \quad (3.75)$$

$$\begin{aligned} S_2 &= \frac{\left(\sum_{i=0}^{n_A} |\delta^{A*} - \hat{\delta}_i^{A*}| + \sum_{i=0}^{n_B} |\delta^{B*} - \hat{\delta}_i^{B*}| \right)}{n^A + n^B} \\ &= wa * PEA_1 + wb * PEB_1. \end{aligned} \quad (3.76)$$

where $wa \equiv n^A / (n^A + n^B)$ and $wb \equiv n^B / (n^A + n^B)$. Both point measures correspond to the previous two cross market pricing error measures, respectively.

Based on the gross risk-free rate of return, the cross market pricing error also can be measured in percentage terms. Therefore, another pair of market segmentation indices can be defined as follows:

$$\begin{aligned} S_3 &= \sqrt{\frac{\sum_{i=0}^{n_A} \left(\frac{\delta^{A*} - \hat{\delta}_i^{A*}}{\delta^{A*}} \right)^2 + \sum_{i=0}^{n_B} \left(\frac{\delta^{B*} - \hat{\delta}_i^{B*}}{\delta^{B*}} \right)^2}{n^A + n^B}} \\ &= \left[wa * \left(\frac{PEA_1}{\delta^{A*}} \right)^2 + wb * \left(\frac{PEB_1}{\delta^{B*}} \right)^2 \right]^{\frac{1}{2}} \end{aligned} \quad (3.77)$$

$$\begin{aligned} S_4 &= \frac{\left(\sum_{i=0}^{n_A} \left| \frac{\delta^{A*} - \hat{\delta}_i^{A*}}{\delta^{A*}} \right| + \sum_{i=0}^{n_B} \left| \frac{\delta^{B*} - \hat{\delta}_i^{B*}}{\delta^{B*}} \right| \right)}{n^A + n^B} \\ &= wa * \left(\frac{PEA_2}{\delta^{A*}} \right) + wb * \left(\frac{PEB_2}{\delta^{B*}} \right) \end{aligned} \quad (3.78)$$

The larger the segmentation index, the more market segmented the two markets are. A zero value of the segmentation index indicates that the two markets are perfectly integrated.

4 DATA AND ESTIMATION METHODS

To estimate the degree of market integration and segmentation and to carry out the alternative test of CCAPM models, two sets of international data are used, namely Taiwan data and U.S. data. Monthly series for the period December, 1980 through November, 1996 are used to estimate the degree of market integration and segmentation both domestically and internationally. Quarterly series for the period January, 1981 through September, 1996 are used to carry out the CCAPM test.

The two major series used are the real per capita consumption expenditure and the classified industry portfolio gross rate of returns. Due to data availability, we use the seasonally unadjusted consumption and price deflator data for the Taiwan equity market, and the corresponding seasonally adjusted data for the U.S. equity market. To be consistent with the empirical finance literature, we use portfolios, instead of individual stocks, because some individual stocks may not be traded for certain periods or have missing observations. Furthermore, we can help reduce the effect of data measurement errors by aggregating individual stock returns into portfolio returns.

4.1 Portfolio Return Data

For the Taiwan portfolio data, the stock price indices reported in the Taiwan Stock Exchange Statistical Data are used to construct Taiwan industry portfolio returns. The seven indices displayed in Table 4.1 are selected to construct seven Taiwan industry portfolio returns. The i -th nominal industry portfolio gross rate of return in period t is

Table 4.1 Stock price index portfolios

Portfolio number	Industry
1	Cement
2	Food
3	Plastics and Chemicals
4	Textiles
5	Electric and Machinery
6	Pulp and Paper
7	Construction

These stock price index data are provided by the Taiwan Stock Exchange, R.O.C.. The industry descriptions can be found in the Standard Industrial Classification System compiled by the Directorate-General of Budget, Accounting, and Statistics, Executive Yuan, R.O.C..

defined as

$$R_{i,t}^P = \frac{SPI_{i,t}}{SPI_{i,t-1}} \quad (4.1)$$

where $SPI_{i,t}$ is the level of the i -th stock price index at the end of the period t .

For the U.S. portfolio returns, we collect stock data from the Center for Research in Security Prices (CRSP) database compiled by the University of Chicago. The three equity exchanges considered in the present study are the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the Nasdaq Stock Exchange (NASDAQ). The CRSP exchange code is used to classify individual stocks into NYSE, AMEX, or NASDAQ trading stocks. For the industry portfolios, common stocks for each exchange and for the U.S. equity market as a whole are grouped according to the first two digits of their standard industrial classification (SIC) code. Twelve SIC grouped industry portfolios are formed for each exchange and for the U.S. equity market as a whole using this two-digit classification. Table 4.2 presents the two-digit industry groupings. The industry groupings follow the classification used by Breeden et al. (1989), Ferson and Harvey (1991), Naranjo and Protopadakis (1997), and others. This industry group-

Table 4.2 SIC industry portfolios

Portfolio number	Two-digit SIC codes	Industry
1	13,29	Petroleum
2	60-69	Finance/real estate
3	25,30,36,37,50,55,57	Consumer durables
4	10,12,14,24,26,28,33	Basic industries
5	1,20,21,54	Food/tobacco
6	15-17,32,52	Construction
7	34,35,38	Capital goods
8	40-42,44,45,47	Transportation
9	46,48,49	Utilities
10	22,23,31,51,53,56,59	Textiles/trade
11	72,73,75,80,82,89	Services
12	27,58,70,78,79	Leisure

The SIC codes and industry descriptions can be found in the Standard Industrial Classification Manual (1987) compiled by the U.S. Government office of Management and Budget. This table is the same as Table A1 in Naranjo and Protopapadakis (1997).

ing has been found to capture significant correlation patterns among stocks. We form 12 value-weighted industry portfolios for each exchange and for the U.S. equity market as a whole, for a total of 48 value-weighted industry portfolios. We also form the corresponding 48 equally-weighted industry portfolios.

For the value-weighted portfolios, the gross rate of return on a particular portfolio in period t (R_t^P) is calculated as the value-weighted average of the returns for the individual stocks in the portfolio:

$$R_t^P = \frac{\sum_{i=1}^{n_P} w_{i,t} r_{i,t}}{\sum_{i=1}^{n_P} w_{i,t}} \quad (4.2)$$

where $r_{i,t}$ is the gross rate of return on the i -th asset in period t ; $w_{i,t}$ is the weight assigned to asset i 's return; and n_P is the number of assets in the portfolio. For each period, relative market value is used to weigh the individual asset return in a given portfolio. The market value ($w_{i,t}$) of a particular asset is defined as the product of its price (P_{t-1}) and its number of shares outstanding (S_{t-1}).

For the equally-weighted portfolios, the value weight $w_{i,t}$ for every stock in the portfolio is set equal to 1 so that the portfolio return is equal to the sum of the individual asset returns divided by the number of assets in the portfolio in a given period.

All constructed nominal portfolio returns are deflated by the domestic seasonally unadjusted price deflator for the nondurables when we estimate the degree of domestic market integration and segmentation. For the purpose of international comparisons, the nominal portfolio returns for the two countries under consideration are first transformed into a common currency unit by means of the exchange rate between these two countries reported by the Financial Statistics Monthly, R.O.C.. Specifically, U.S. dollar returns $R_t^{\$}$ are transformed into New Taiwan (NT) dollar returns $R_t^{NT\$}$ using the following expression:

$$R_t^{NT\$} = R_t^{\$}(e_t/e_{t-1}) \quad (4.3)$$

where e_t is the NT dollar price per unit of US dollar at the end of period t . Analogously, NT dollar returns are transformed into US dollar returns using the following expression:

$$R_t^{\$} = R_t^{NT\$}(e_{t-1}/e_t). \quad (4.4)$$

The transformed nominal common currency portfolio returns are then deflated by the common price deflator for the nondurables chosen from either of the two countries' price deflator.

4.2 Consumption Data

Two different measures of consumption expenditures are considered to carry out the asset pricing test: real per capita nondurables (ND) and real per capita nondurables plus services (NDS). For Taiwan, however, these measures are reported only annually by Directorate-General of Budget, Accounting and Statistics (DGBAS), Executive Yuan, Republic of China. On the other hand, the more detailed real consumption expenditure

Table 4.3 Classification of seasonally unadjusted consumption data

Consumption category	Expenditure Component
Durables	Transport and communication
	Furniture, furnishings and household equipment
Nondurables	Food
	Clothing and footwear
	Fuel and power
	Tobacco
	Beverages
Services	Household operation
	Rent and Water charges
	Recreation, educational and cultural services
	Medical care and health expenses
	Miscellaneous

The expenditure components can be found in Quarterly National Income Statistics in Taiwan Area, The Republic of China compiled by the Directorate-General of Budget, Accounting and Statistics, Executive Yuan. This table is similar to Table A1 in Miron (1986).

components are reported quarterly by Quarterly National Economic Trends, DGBAS. Therefore, some criterion is required to construct the two series ND and NDS. The criterion used to classify the consumption expenditure components into ND and NDS is similar to that of Miron (1986) and displayed in Table 4.3. The only difference is the transport and communication component. For U.S. data, transportation and auto parts are classified into services and durables, respectively. For Taiwan data, both components are classified into durables. Observing the annual data reveals that the transport subcomponent only has a very small proportion of the transport and communication component. Therefore, it is reasonable to classify the transport and communication component into durables.

To obtain real nondurables plus services expenditures, we sum the constant-dollar expenditure on nondurables and the constant-dollar expenditure on services multiplied by the relative price of services in terms of nondurables. The total population provided by the Ministry of Interior, R.O.C., is then used to divide the real ND and NDS series

to obtain the per capita base data.

For U.S. consumption data, the real per capita nondurables and services and the price deflator in 1992 dollars are collected from the Survey of Current Business, Department of Commerce. We also use the relative price deflator to construct the real per capita nondurables plus services series.

4.3 Estimation and Testing Procedures

Suppose there is a total of N assets in the domestic financial market. To measure and test the degree of domestic market integration and segmentation, we form two artificial markets A and B with n^A and n^B assets selected from the N assets in the domestic financial market, respectively. Specifically, the two artificial markets are formed following the two steps below:

Step 1: Randomly select n^A assets without replacement from the N assets in the domestic financial market.

Step 2: Randomly select n^B assets from the rest of $(N - n^A)$ assets in the domestic financial market.

If the total number of possible combinations for the two artificial markets A and B to be formed from the N assets is less than a prespecified number, say 1,000 in the present study, we choose to measure the degree of domestic market integration and segmentation for each possible combination of the two artificial markets. Otherwise, we repeat the randomization procedure summarized in steps 1 and 2 for 500 random draws to obtain 500 possible combinations for the two artificial markets.

For each chosen combination of the two artificial markets, we first use the optimal value of the divergence metric D^* between these two artificial markets to estimate the degree of market integration.

Next, we use the segmentation indices S_1 through S_4 defined in expressions (3.75)

through (3.78), respectively, to measure the degree of market segmentation.

Finally, we use the chi-squared statistics corresponding to expression (3.66) to test for the integration hypothesis that the two artificial markets A and B are perfectly integrated:

$$(T/2)D^* \sim \chi_{(T-n^A-n^B+1)}^2 \quad (4.5)$$

where T (corresponding to S in expression (3.66)) is the number of periods (i.e., observations) in the sample. Note that $n^A = n_A + 1$ and $n^B = n_B + 1$ because we represent the linear pricing constraints in the form of excess returns relative to the return of the base asset. Therefore, the asymptotic chi-squared distribution has $(T - n^A - n^B + 1)$ degrees of freedom.

We collect the estimates of the degree of domestic market integration and segmentation for each chosen combination of the artificial markets, and then average such estimates over all chosen combinations of artificial markets. The resulting mean values represent the indices of the degree of domestic market integration and segmentation in the present study. We run this procedure separately for each pair (n^A, n^B) formed from the N assets in the domestic financial market.

The purpose of the above procedure is to test the hypothesis that the N -asset domestic financial market is perfectly integrated. For an integrated N -asset domestic financial market, the asset pricing model predicts that any pair of the artificial markets formed from this domestic market should be perfectly integrated (Chen and Knez, 1995). Based on the above pair-wise testing procedure, we can conclude that the N -asset domestic financial market is not perfectly integrated if any chosen combination of any pair of the artificial markets formed from the domestic market is not integrated. Otherwise, the domestic integration hypothesis cannot be rejected.

We apply the same procedure to examine the degree of international market integration and segmentation. The only difference is the randomization procedure. Suppose

there are two countries with N_A and N_B assets in their domestic financial markets, respectively. In the international case, we form the two artificial markets by randomly selecting n^A and n^B assets from the N_A -asset and N_B -asset international financial markets, respectively.

For the purpose of international comparisons, two alternative tests are conducted. In the first test, the nominal U.S. dollar portfolio returns for the U.S. equity market are transformed into the common NT dollar returns. Then, all nominal portfolio returns for both Taiwan and U.S. equity markets are deflated by the common Taiwan price deflator for the nondurables. In the second test, the nominal NT dollar portfolio returns for the Taiwan equity market also are transformed into the common U.S. dollar returns. All nominal portfolio returns for both Taiwan and U.S. equity markets are then deflated by the common U.S. price deflator for the nondurables.

5 EMPIRICAL RESULTS OF MARKET INTEGRATION AND SEGMENTATION

5.1 Empirical Results of Market Integration

Table 5.1 reports the mean values of the estimates of the divergence metrics for each pair (n^A, n^B) of the artificial markets formed from the seven-portfolio Taiwan equity market. The mean value of the estimates of the divergence metrics is effectively 0.000 for all possible pairs of the artificial markets. The empirical results indicate that none of the chi-squared test statistics (not reported in the present study) for any chosen combination of any pair of the artificial markets rejects the integration hypothesis at any reasonable significance level. Therefore, the domestic integration hypothesis for the Taiwan equity market cannot be rejected.

The mean values of the estimates of the divergence metrics for each pair of the artificial markets formed from the-twelve-SIC-industry-portfolio U.S. equity market are reported in Table 5.2. For both value-weighted and equally-weighted scenarios, the mean

Table 5.1 Estimation of the domestic market integration measure: Taiwan equity market

n^B	n^A	
	2	3
2	0.000	
3	0.000	0.000
4	0.000	0.000
5	0.000	

Table 5.2 Estimation of the domestic market integration measure: US equity market

Value-weighted SIC industry portfolios

n^B	n^A				
	2	3	4	5	6
2	0.000				
3	0.000	0.000			
4	0.000	0.000	0.000		
5	0.000	0.000	0.000	0.000	
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	
8	0.000	0.000	0.000		
9	0.000	0.000			
10	0.000				

Equally-weighted SIC industry portfolios

n^B	n^A				
	2	3	4	5	6
2	0.000				
3	0.000	0.000			
4	0.000	0.000	0.000		
5	0.000	0.000	0.000	0.000	
6	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	
8	0.000	0.000	0.000		
9	0.000	0.000			
10	0.000				

value of the estimates of the divergence metrics is effectively 0.000 for all possible pairs of the artificial markets. Empirical results indicate that, in either scenario, none of the chi-squared test statistics for any chosen combination of any pair of the artificial markets rejects the integration hypothesis at any reasonable significance level. Therefore, the domestic integration hypothesis for the U.S. equity market cannot be rejected.

To further investigate the domestic U.S. equity market integration, we examine the degree of interexchange market integration for three U.S. exchanges: NYSE, AMEX, and

NASDAQ. The pair that selects all assets in any two exchanges to form the artificial markets is referred to as interexchange pair. Table 5.3 reports the mean values of the estimates of the divergence metrics for the interexchange pairs. For both value-weighted and equally-weighted scenarios, the mean value of the estimates of the divergence metrics between any two exchanges is effectively 0.000. In general, these interexchange divergence metrics are negligible. These empirical results provide further evidence that the U.S. equity market is domestically integrated.

Table 5.3 Estimation of the interexchange market integration measure: NYSE, AMEX, and NASDAQ

Value-weighted SIC industry portfolios		
	NYSE	AMEX
AMEX	0.000	
NASDAQ	0.000	0.000

Equally-weighted SIC industry portfolios		
	NYSE	AMEX
AMEX	0.000	
NASDAQ	0.000	0.000

The empirical results of the international market integration measures for the NT dollar return scenario are reported in Table 5.4. We refer to the pair that selects all assets in these two international equity markets to form the artificial markets as cross-market pair. In Table 5.4, the mean values of the estimates of the divergence metrics for the cross-market pair are effectively 0.000 for the value-weighted and equally-weighted cases. For both value-weighted and equally-weighted cases, the mean values of the estimates of the divergence metrics for any other two artificial markets also are effectively 0.000.

The U.S. dollar return scenario of the international market integration measures has the same empirical results as those in Table 5.4. The mean values of the estimates for the cross-market pair and all other pairs of the artificial markets are effectively 0.000 for

Table 5.4 Estimation of the international market integration measure:
Taiwan and U.S. equity markets (NT dollars)

Taiwan versus U.S. value-weighted portfolios

n^B	n^A					
	2	3	4	5	6	7
2	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000

Taiwan versus U.S. equally-weighted portfolios

n^B	n^A					
	2	3	4	5	6	7
2	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000

both value-weighted and equally-weighted cases.

All of the above figures are negligible in both economic and statistical terms, indicating that these two international equity markets are perfectly integrated. Formally, none of the chi-squared test statistics for each chosen case rejects the integration hypothesis. Therefore, the international integration hypothesis for Taiwan and U.S. equity markets cannot be rejected.

In summary, the empirical results provide evidence that at the industry portfolio level, both Taiwan and U.S. equity markets are integrated not only domestically, but also internationally.

5.2 Empirical Results of Market Segmentation

Table 5.5 reports the mean values of the estimates of four segmentation indices for each pair (n^A, n^B) of the artificial markets formed from the seven-portfolio Taiwan equity market. Under the perfect integration hypothesis, the value of the cross-market mispricing (i.e., the segmentation indices) is equal to zero. In practice, the more integrated the two markets are, the closer to zero the market segmentation index should be. For the Taiwan equity market, the cross-market mispricings of the mean estimates for four segmentation indices are effectively 0.000 for all possible pairs of the artificial markets. They are generally very small. These results indicate that the domestic Taiwan equity market is not segmented, which is consistent with the testing result that the Taiwan equity market is domestically integrated.

The twelve-SIC-industry-portfolio U.S. equity market has the same empirical results as those in the Taiwan equity market. For both value-weighted and equally-weighted scenarios, the cross-market mispricings for the four segmentation indices are effectively 0.000 for all possible pairs of the artificial markets. These results indicate that the domestic U.S. equity market is not segmented, which also is consistent with the testing

Table 5.5 Estimation of domestic market segmentation measures: Taiwan equity market

S_1		
n^B	n^A	
	2	3
2	0.000	
3	0.000	0.000
4	0.000	0.000
5	0.000	

S_2		
n^B	n^A	
	2	3
2	0.000	
3	0.000	0.000
4	0.000	0.000
5	0.000	

S_3		
n^B	n^A	
	2	3
2	0.000	
3	0.000	0.000
4	0.000	0.000
5	0.000	

S_4		
n^B	n^A	
	2	3
2	0.000	
3	0.000	0.000
4	0.000	0.000
5	0.000	

result that the U.S. equity market is domestically integrated.

To further investigate the domestic U.S. market segmentation, we also examine the degree of market segmentation for the stock exchanges NYSE, AMEX, and NASDAQ. For both value-weighted and equally-weighted scenarios, the four segmentation indices of the interexchange pairs are effectively 0.000. The negligible value of these segmentation indices indicates that the three stock exchanges are not segmented from each other. These empirical results further verify the testing result that the U.S. equity market is domestically integrated.

The empirical results of the international market segmentation measures for the value-weighted NT dollar return scenario are reported in Table 5.6. The cross-border mispricings of the mean estimates for four segmentation indices are effectively 0.000 for all possible pairs of the artificial markets. For the equally-weighted NT dollar return scenario, the cross-border mispricings of the mean estimates for the four segmentation indices also are effectively 0.000. Again, these figures are negligible.

The U.S. dollar return scenario of the international market segmentation measures has the same empirical results as those in the NT dollar return scenario. For both value-weighted and equally-weighted cases, the cross-border mispricings of the mean estimates for the four segmentation indices are effectively 0.000.

From the above analysis, the cross-border mispricing is generally negligible, indicating that the Taiwan and U.S. equity markets are not internationally segmented. This result also is consistent with the testing result that these two markets are internationally integrated.

5.3 Summary

We use the highly aggregated portfolio returns for Taiwan and U.S. equity markets to investigate the degree of market integration and segmentation domestically and

Table 5.6 Estimation of international market segmentation measures:
Taiwan and U.S. equity markets (NT dollars)

Taiwan versus U.S. value-weighted portfolios: S_1

n^B	n^A					
	2	3	4	5	6	7
2	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000

Taiwan versus U.S. value-weighted portfolios: S_2

n^B	n^A					
	2	3	4	5	6	7
2	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000

Table 5.6 (Continued)

Taiwan versus U.S. value-weighted portfolios: S_3

n^B	n^A					
	2	3	4	5	6	7
2	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000

Taiwan versus U.S. value-weighted portfolios: S_4

n^B	n^A					
	2	3	4	5	6	7
2	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.000

internationally. The empirical results indicate that Taiwan and U.S. equity markets are integrated domestically and internationally because both the integration (i.e., the divergence metrics) and segmentation indices are very small and none of the chi-squared test statistics is statistically significant. These results are consistent with most existing empirical work that uses indices for market integration test (Stulz, 1994). At the individual stock level, we impose more linear pricing constraints on the structure of a financial market. The greater the number of constraints on the structure of a financial market, the smaller the set of admissible risk-neutral probability measures for the market, implying larger integration and segmentation indices between any two financial markets. Therefore, the empirical results are also consistent with our intuition that the markets are more likely to be integrated at the aggregated portfolio level than at the individual stock level.

6 AN ALTERNATIVE TEST OF THE CCAPM MODEL

Suppose that the asset data set passes the integration test and that the sample information contained in the data set can be transformed into n linear pricing constraints (3.22). How do we implement an alternative entropic test of a candidate CCAPM model?

6.1 An Entropic Testing Procedure for CCAPM Models

To carry out an entropic test of a candidate CCAPM model, we need to give empirical content to expression (3.56). In other words, we need to specify the functional form of the felicity function $U(\cdot)$ for the stochastic discount factor and to estimate the actual probability measure Π_j .

We use the uniform distribution to estimate the actual probability measure, i.e., $\Pi_j = 1/S$, because the Shannon entropy is maximized when the probability measure follows a uniform distribution. We also use the following power utility function to estimate the stochastic discount factor:

$$U(C) = \frac{C^{1-\gamma}}{(1-\gamma)} \quad (6.1)$$

where γ is the coefficient of relative risk aversion. The power form is the most popular felicity function in empirical finance. Power felicity greatly simplifies the estimation problem because the testing method used in the present study requires stationary variables. Power felicity function allows us to work with the stationary consumption ratio C_{t+1}/C_t , instead of the nonstationary consumption level C_t . Under certain conditions,

we can aggregate power-felicity agents with different wealth levels and different levels of relative risk aversion into a single “representative” agent with power felicity (Altug and Labadie, 1994; p.22). Furthermore, different felicity functions with the same relative risk aversion yield almost identical decisions (Kallberg and Ziemba, 1984).

The stochastic discount factor implied by the power utility CCAPM model can be derived as a function of the consumption ratio between time $t + 1$ and time t :

$$M_{t+1} = \delta(C_{t+1}/C_t)^{-\gamma}. \quad (6.2)$$

Under the maintained hypothesis of stationarity, expressions (3.9) and (6.2) allow us to estimate the risk-neutral probability measure implied by the power felicity CCAPM model by the following expression:

$$P_t = \frac{\frac{1}{T}}{\frac{1}{T} \sum_{t=1}^T M_t} M_t = \frac{(C_t/C_{t-1})^\gamma}{\sum_{t=1}^T (C_t/C_{t-1})^\gamma}. \quad (6.3)$$

Consider a financial market that has passed the integration test. The sample information contained in $n + 1$ assets used in the empirical test can be transformed into the form of n linear pricing constraints (3.22). Given a particular value of the coefficient of relative risk aversion, we use expression (6.3) to estimate the prior risk-neutral probability measure q implied by the power felicity CCAPM model. Then, the asymptotic chi-squared distribution in expression (3.57) can be used to test for the power felicity CCAPM model:

$$2TH^* \sim \chi_{(T-n-1)}^2 \quad (6.4)$$

where T (corresponding to S in expression (3.57)) is the number of periods (observations) in the sample. We repeat the entropic testing procedure for each reasonable value of the coefficient of relative risk aversion specified in the present study.

The proposed alternative entropic testing procedure for the candidate CCAPM model can be summarized as follows:

Step 1: For a given set of values of the parameters in the candidate CCAPM model within the reasonable range of parameter space, employ expression (6.3) to estimate the implied risk-neutral probability measure q .

Step 2: Based on the cross entropy minimization problem of the divergence between any two risk-neutral probability measures, treat the risk-neutral probability measure q calculated in step 1 as the prior to find the risk-neutral probability measure P^* used to price all assets in an integrated market.

Step 3: Based on the asymptotic chi-squared distribution (6.4), test whether the probability measure q implied by the candidate CCAPM model is statistically significantly different from P^* estimated from the corresponding asset data set. If the two probability measures are significantly different, the candidate CCAPM model fails. Otherwise, it is not rejected.

Step 4: Repeat steps 1 to 3 until all prespecified parameters of the candidate CCAPM model have been used.

6.2 Empirical Results of the CCAPM Test

We have verified the assumption of CCAPM models that all assets used in the econometric analysis are in a perfectly integrated financial market by performing tests on the domestic integration hypothesis for Taiwan and U.S. equity markets in section 3.4.2. Therefore, we can confidently test the power felicity CCAPM model. The reasonable range of the coefficient of relative risk aversion considered in the present study is between 0.5 and 9.0 (Kocherlakota, 1996). Given a particular value of the coefficient of relative risk aversion, we use two measures of consumption expenditures, namely nondurables and nondurables plus services, to estimate the prior information q implied by the power felicity CCAPM model.

The results of the cross entropy metric estimates for the Taiwan equity market are

reported in Table 6.1. Numbers within parenthesis are the corresponding chi-squared test statistics. For the nondurables case, the point estimates of the cross entropy metric range only from 0.024 to 0.060. For the nondurables plus services case, the point estimates range only from 0.023 to 0.086. Comparison of columns 2 and 3 reveals that the inclusion of services component has little effect on their values. The point estimates of the cross entropy metric also vary little with the value of the coefficient of relative risk aversion. The critical value for the chi-squared statistic with 55 degrees of freedom at ten percent significance level is 69.918. In all cases, the calculated chi-squared test statistics are very low compared to the above critical value. Therefore, the power felicity CCAPM model cannot be rejected for the Taiwan equity market.

Table 6.2 reports the results of the cross entropy metric estimates and chi-squared test statistics for the U.S. equity market. Two sets of SIC-industry-portfolio returns, namely value-weighted and equally-weighted, are used to perform the power felicity CCAPM model. For the value-weighted scenario, the minimum and maximum values of the point estimates of the cross entropy metric for nondurables are 0.111 and 0.113, respectively. The minimum and maximum value of the point estimates for nondurables plus services are 0.108 and 0.113, respectively. Comparison of columns 2 and 3 in Table 6.2 reveals that the point estimates of the cross entropy metric display little variation with either the coefficient of relative risk aversion or the type of consumption expenditure. In the corresponding equally-weighted scenario, the results display the same pattern as those for the value-weighted scenario. The critical value for the chi-squared statistic with 50 degrees of freedom at ten percent significant level is 64.295. In all cases, the estimated chi-squared test statistics are very low compared to the above critical value. Therefore, the power felicity CCAPM model also cannot be rejected for the U.S. equity market.

In summary, the power felicity CCAPM model cannot be rejected for Taiwan and U.S. equity markets at any conventional significance level because the estimated chi-squared test statistics are relatively very low.

Table 6.1 Power felicity CCAPM test:
Taiwan equity market

RRA	ND	NDS
0.5	0.024 (2.980)	0.023 (2.931)
1.0	0.024 (2.988)	0.023 (2.917)
1.5	0.024 (3.021)	0.023 (2.955)
2.0	0.024 (3.081)	0.024 (3.049)
2.5	0.025 (3.169)	0.025 (3.201)
3.0	0.026 (3.286)	0.027 (3.414)
3.5	0.027 (3.435)	0.029 (3.689)
4.0	0.029 (3.618)	0.032 (4.028)
4.5	0.030 (3.835)	0.035 (4.432)
5.0	0.032 (4.087)	0.039 (4.901)
5.5	0.035 (4.377)	0.043 (5.435)
6.0	0.037 (4.706)	0.048 (6.033)
6.5	0.040 (5.073)	0.053 (6.695)
7.0	0.044 (5.481)	0.059 (7.417)
7.5	0.047 (5.929)	0.065 (8.199)
8.0	0.051 (6.418)	0.072 (9.038)
8.5	0.055 (6.949)	0.079 (9.930)
9.0	0.060 (7.521)	0.086 (10.874)

Table 6.2 Power felicity CCAPM test:
U.S. equity market

RRA	Value weighted		Equally weighted	
	ND	NDS	ND	NDS
0.5	0.113 (14.290)	0.113 (14.265)	0.111 (13.967)	0.111 (13.928)
1.0	0.113 (14.269)	0.113 (14.221)	0.111 (13.983)	0.110 (13.905)
1.5	0.113 (14.249)	0.113 (14.177)	0.111 (13.999)	0.110 (13.882)
2.0	0.113 (14.229)	0.112 (14.134)	0.111 (14.016)	0.110 (13.860)
2.5	0.113 (14.209)	0.112 (14.091)	0.111 (14.033)	0.110 (13.838)
3.0	0.113 (14.190)	0.112 (14.048)	0.112 (14.050)	0.110 (13.816)
3.5	0.113 (14.171)	0.111 (14.006)	0.112 (14.068)	0.110 (13.794)
4.0	0.112 (14.152)	0.111 (13.964)	0.112 (14.086)	0.109 (13.773)
4.5	0.112 (14.133)	0.111 (13.923)	0.112 (14.104)	0.109 (13.753)
5.0	0.112 (14.115)	0.110 (13.882)	0.112 (14.123)	0.109 (13.732)
5.5	0.112 (14.097)	0.110 (13.841)	0.112 (14.141)	0.109 (13.712)
6.0	0.112 (14.079)	0.110 (13.801)	0.112 (14.160)	0.109 (13.692)
6.5	0.112 (14.061)	0.109 (13.761)	0.113 (14.180)	0.109 (13.673)
7.0	0.112 (14.044)	0.109 (13.722)	0.113 (14.199)	0.108 (13.654)
7.5	0.111 (14.027)	0.109 (13.683)	0.113 (14.219)	0.108 (13.635)
8.0	0.111 (14.010)	0.108 (13.644)	0.113 (14.240)	0.108 (13.617)
8.5	0.111 (13.994)	0.108 (13.606)	0.113 (14.260)	0.108 (13.599)
9.0	0.111 (13.977)	0.108 (13.568)	0.113 (14.281)	0.108 (13.581)

7 CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

An interesting and important question asked in the present study is whether an emerging equity market is integrated domestically. Starting from this primary concern, the main purpose of the present study is to measure the degree of market integration and segmentation for an emerging equity market (Taiwan), as well as a mature equity market (U.S.), both domestically and internationally. The other purpose of the present study is to propose an alternative entropic test of conventional CCAPM models.

The conventional method in international integration tests either is subject to the joint hypothesis test problem or lacks of sampling distribution needed to make inferences about the integration hypothesis. To circumvent both problems, we introduce a new tool, the nonparametric entropy pricing theory, to test the integration hypothesis.

The entropy pricing framework is based on the risk-neutral representation. It automatically imposes the positivity constraint on the artificial risk-neutral probability measure. The asymptotic chi-squared distribution within this entropic framework allows us to statistically test for the integration hypothesis and for a candidate CCAPM model.

Both monthly and quarterly consumption and return data for Taiwan and U.S. equity markets are used in empirical analysis. At the highly aggregated industry portfolio level, we find that both equity markets are integrated domestically and internationally. The small value of the estimated segmentation indices provides evidence that the two equity

markets are not segmented, either domestically or internationally. We also find that the power felicity CCAPM model cannot be rejected separately for these two equity markets.

Empirically, the proposed nonparametric entropic approach to market integration could be applied to any two markets within or across countries. Typical potential choices could be bond, forward contract, futures, and options markets. On the other hand, the proposed entropic power utility CCAPM test could also be applied to other types of CCAPM models, e.g., generalized expected utility, habit formation, and consumption durability models. It could also be extended to test for other framework of asset pricing models, e.g., conventional CAPM and APT asset pricing models.

Another interesting application could be conducting the empirical analysis for a large number of assets. For example, if we use individual stock (instead of industry portfolio) returns for the integration test, entropy pricing theory predicts that the optimal value of the divergence metric will be increased. Therefore, at the individual stock level, the market integration hypothesis might be rejected.

A major drawback to the proposed entropic approach may be that the optimal probability measures will tend to have low entropy (high information) for the events in the individual markets. In this case, we could use the parametric overidentifying test developed by Kitamura and Stutzer (1997) to overcome this problem. By imposing additional linear pricing constraints implied by another financial market structure, we could use a similar chi-squared test statistics to test for market integration.

APPENDIX

THE SOLUTION TO THE CROSS ENTROPY PROBLEM

This appendix derives the solution to the primal and dual cross entropy minimization problem (3.41) through (3.44) discussed in section 3.3.1.

The primal minimum cross entropy problem

To solve the minimization problem (3.41)-(3.44), we form the following Lagrangian function

$$\mathcal{L} = \sum_{j=1}^S P_j \ln(P_j/q_j) + \sum_{i=1}^n \lambda_i \left(- \sum_{j=1}^S P_j X_{ij} \right) + \lambda_0 \left(1 - \sum_{j=1}^S P_j \right) \quad (\text{A.1})$$

with the first order necessary conditions

$$0 = \mathcal{L}_{P_j} = \ln(P_j/q_j) + 1 - \sum_{i=1}^n \lambda_i x_{ij} - \lambda_0, \quad j = 1, \dots, S \quad (\text{A.2})$$

$$0 = \mathcal{L}_{\lambda_i} = - \sum_{j=1}^S P_j X_{ij} \quad (\text{A.3})$$

$$0 = \mathcal{L}_{\lambda_0} = 1 - \sum_{j=1}^S P_j. \quad (\text{A.4})$$

Following the same procedure used in the maximum entropy problem, we derive the solution to the minimization problem

$$\begin{aligned} \tilde{P}_j = \bar{P}_j(\tilde{\lambda}) &= \frac{q_j e^{\sum_{i=1}^n \tilde{\lambda}_i X_{ij}}}{\sum_{j=1}^S q_j e^{\sum_{i=1}^n \tilde{\lambda}_i X_{ij}}} \\ &= \frac{q_j e^{\sum_{i=1}^n \tilde{\lambda}_i X_{ij}}}{\Omega(\tilde{\lambda})} \end{aligned} \quad (\text{A.5})$$

where $\Omega(\tilde{\lambda}) \equiv \Omega(\tilde{\lambda}_1, \dots, \tilde{\lambda}_n) = \sum_{j=1}^S q_j e^{\sum_{i=1}^n \tilde{\lambda}_i X_{ij}}$ is a function of the Lagrange multipliers on the constraint (3.42).

The cross entropy minimization problem always has a unique solution because the objective is a strictly convex function on a convex set. The solution \tilde{P} satisfies the additivity constraint (3.43), and \tilde{P}_j is strictly positive.

As previously mentioned, to increase computational efficiency, the risk-neutral probability measure \tilde{P} can be recovered by finding $\tilde{\lambda}$ from the unconstrained dual cross entropy problem.

The unconstrained dual cross entropy problem

Recall, from expression (A.5), that

$$\ln(P_j/q_j) = \sum_{i=1}^n \lambda_i X_{ij} - \ln\left(\sum_{j=1}^S q_j e^{\sum_{i=1}^n \lambda_i X_{ij}}\right). \quad (\text{A.6})$$

Analogous to the unconstrained dual maximum entropy objective, the dual cross entropy objective may be formulated, from expressions (A.1) and (A.5), as

$$\begin{aligned} L(\lambda) &= \sum_{j=1}^S P_j \left[\sum_{i=1}^n \lambda_i X_{ij} - \ln\left(\sum_{j=1}^S q_j e^{\sum_{i=1}^n \lambda_i X_{ij}}\right) \right] - \sum_{i=1}^n \lambda_i \sum_{j=1}^S P_j X_{ij} \\ &= - \sum_{j=1}^S P_j \ln\left(\sum_{j=1}^S q_j e^{\sum_{i=1}^n \lambda_i X_{ij}}\right). \end{aligned}$$

Since $\ln(\sum_{j=1}^S q_j e^{\sum_{i=1}^n \lambda_i X_{ij}})$ is fixed and $\sum_{j=1}^S P_j = 1$, we obtain

$$\begin{aligned} L(\lambda) &= -\ln\left(\sum_{j=1}^S q_j e^{\sum_{i=1}^n \lambda_i X_{ij}}\right) \\ &= -\ln\Omega(\lambda) \equiv -M(\lambda). \end{aligned} \quad (\text{A.7})$$

The following claim shows that the unconstrained maximization problem has a unique global solution for λ .

Claim 2: The dual objective $-M(\lambda)$ is strictly concave in λ and has a unique global solution for λ .

Proof: It suffices to prove that the Hessian matrix $M_{\lambda\lambda}^-$, of the dual objective $-M(\lambda)$ is everywhere negative definite assuring a unique global solution for λ . Note that the l -th gradient of the dual objective is

$$M_{\lambda_l}^- = - \sum_{j=1}^S X_{lj} P_j. \quad (\text{A.8})$$

The l -th row, l' -th column element of the Hessian matrix of the dual objective is

$$\begin{aligned} M_{\lambda_l \lambda_{l'}}^- &= - \sum_{j=1}^S X_{lj} \frac{\partial P_j}{\partial \lambda_{l'}} \\ &= - \sum_{j=1}^S X_{lj} \left[\frac{1}{\Omega(\lambda)} q_j X_{l'j} e^{-\sum_{i=1}^n \lambda_i X_{ij}} + q_j e^{\sum_{i=1}^n \lambda_i X_{ij}} \frac{-1}{\Omega(\lambda)\Omega(\lambda)} \right. \\ &\quad \left. \sum_{j=1}^S q_j X_{l'j} e^{\sum_{i=1}^n \lambda_i X_{ij}} \right] \\ &= - \sum_{j=1}^S X_{lj} [X_{l'j} P_j - P_j \sum_{j=1}^S X_{l'j} P_j]. \end{aligned}$$

Since $E(\tilde{X}_i) = \sum_{j=1}^S X_{ij} P_j$ is fixed, we obtain

$$M_{\lambda_l \lambda_{l'}}^- = -(E(\tilde{X}_l \tilde{X}_{l'}) - E(\tilde{X}_l)E(\tilde{X}_{l'})) = -Cov(\tilde{X}_l, \tilde{X}_{l'}). \quad (\text{A.9})$$

Therefore, the $n \times n$ Hessian matrix of the dual objective is the following matrix:

$$M_{\lambda\lambda}^- = - \begin{bmatrix} Var(\tilde{X}_1) & Cov(\tilde{X}_1, \tilde{X}_2) & \cdots & Cov(\tilde{X}_1, \tilde{X}_n) \\ Cov(\tilde{X}_1, \tilde{X}_2) & Var(\tilde{X}_2) & \cdots & Cov(\tilde{X}_2, \tilde{X}_n) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(\tilde{X}_1, \tilde{X}_n) & Cov(\tilde{X}_2, \tilde{X}_n) & \cdots & Var(\tilde{X}_n) \end{bmatrix}. \quad (\text{A.10})$$

As $P(\lambda)$ is strictly positive and the variance-covariance matrix is positive definite, the Hessian matrix $M_{\lambda\lambda}^-$, is everywhere negative definite assuring a unique global solution for λ .

Maximizing the dual objective $-M(\lambda)$ amounts to minimizing the objective $M(\lambda)$. Therefore, to recover $\tilde{P}(\tilde{\lambda})$, we can first find $\tilde{\lambda}$ by solving the following unconstrained minimization problem:

$$\min_{\lambda} M(\lambda) \tag{A.11}$$

which in turn yields $\tilde{P}(\tilde{\lambda})$. As previously mentioned, this increases computational efficiency, and is assured by the property of uniqueness of the global solutions for the primal and dual problems.

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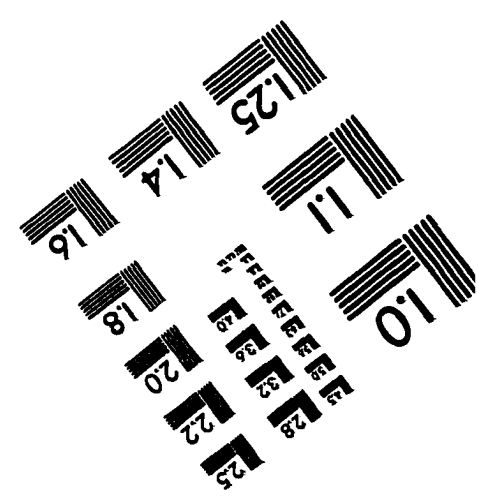
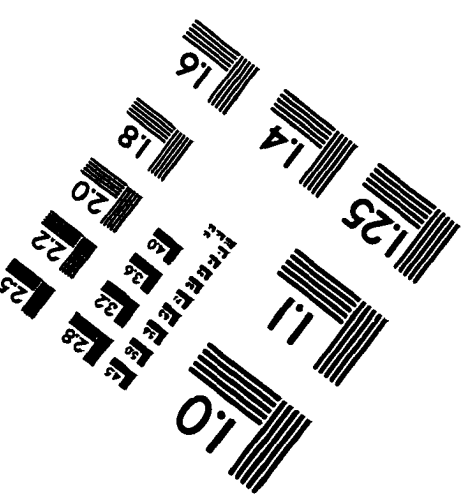
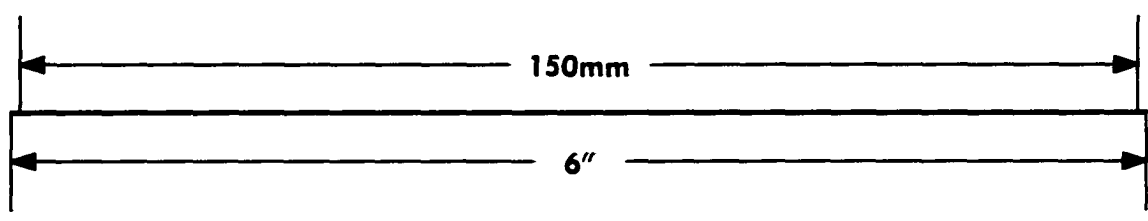
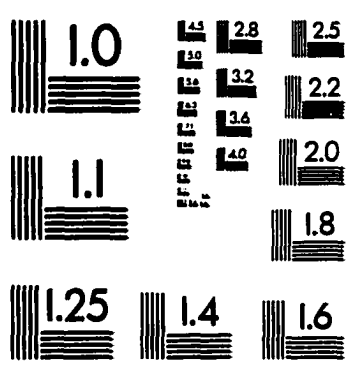
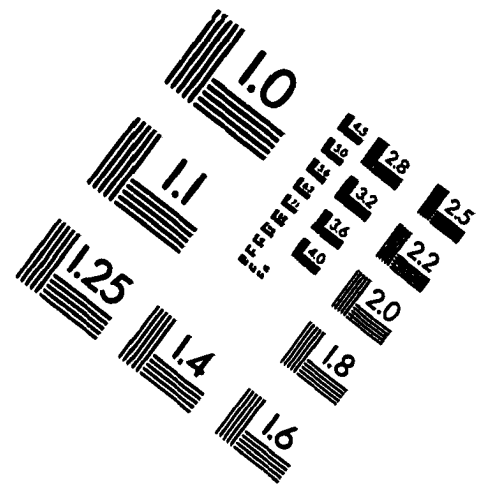
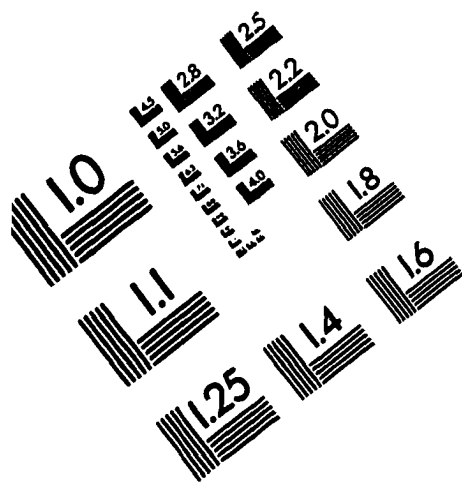
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IMAGE EVALUATION TEST TARGET (QA-3)



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